CODING THEORY, MATH 567
FINAL TAKE HOME EXAM

DUE: APRIL 28

You should work on this final exam individually.

(1) (a) Do §5.2, problem 13.
    (b) Do §5.2, problem 14.

(2) (a) Do §6.2, problem 12. Prove your answer!
    (b) Do §6.2, problem 13.

(3) (a) Write $x^{18} - 1$ as product of cyclotomic polynomials. Determine each cyclotomic polynomial explicitly.
    (b) Factor each of these cyclotomic polynomials into irreducible polynomials over the field $\mathbb{F}_7$. This yields a factorization of $x^{18} - 1$. Find the irreducible factors explicitly. (If you cannot find the irreducible factors explicitly, try to determine at least the degrees of the irreducible factors.)
    (c) How many linear cyclic codes of length 18 over $\mathbb{F}_7$ exist?

(4) (a) Suppose that C is a linear cyclic code of length n over $\mathbb{F}_q$. Let $\mathbf{1} = (111 \cdots 1)$. Prove that $\mathbf{1} \in C$ or $\mathbf{1} \in C^{\perp}$.
    (b) Show that if C is self-dual, then $\gcd(n, q) \neq 1$.
    (c) Prove that a nonzero self-orthogonal cyclic code exists if and only if $q^r \not\equiv -1 \bmod n$ for all positive integers r.

(5) Do §8.1, problem 4.

Date: Winter 2004.