CODING THEORY, MATH 567
PROBLEM SET 5
DUE: WEDNESDAY, APRIL 11

(1) §4.5, Exercise 23.
(2) §4.5, Exercise 25.
(3) §5.2, Exercise 10.
(4) Let 
\[ W_C(s) = A_0 + A_1s + A_2s^2 + \cdots + A_ns^n \]
be the weight enumerator of a linear \([n, k]\) code over \(F_q\).
(a) Let 
\[ W_{C^\perp}(s) = A_0^\perp + A_1^\perp s + A_2^\perp s^2 + \cdots + A_n^\perp s^n \]
be the weight enumerator of the dual code \(C^\perp\). Express \(A_1^\perp\)
in terms of \(A_0, A_1, \ldots, A_n\) using the MacWilliams identity.
(b) Suppose that every position \(i\), there exists a codeword \(x \in C\) such that \(x\) has a nonzero entry at position \(i\). Show that 
\(A_1^\perp = 0\).
(c) Under the assumption of (b), give an explicit formula for
the average weight
\[ \frac{1}{|C|} \sum_{x \in C} w(x) \]
of a codeword (in terms of \(n, k, q\) say).
(5) Write down a generator matrix of a \([6, 3, 4]\) MDS-code over \(F_5\).
(6) §6.1, Exercise 16.
(7) (Bonus) We study the Hamming code \(C = H_q(r)\). This is an
\([n, n - r, 3]\) code over the field \(F_q\) where \(n = (q^r - 1)/(q - 1)\).
(a) Prove that the weight enumerator \(W_{C^\perp}(s)\) of the dual code
is equal to
\[ 1 + (q^r - 1)s^{q^{r-1}} \]
(b) Use (a) to write down a formula for \(W_C(s)\), the weight
enumerator of \(C\).

\textit{Date: Winter 2007.}