MATH 594, WINTER 2006, PROBLEM SET 2

DUE: 1/30/2006

Warm-up (not to be handed in)

[DF], §4.4, Exercises 1, 3, 5, 6 (done in class), 13,19 (after you have done 18, see below), §4.5, Exercise 1, 3, 4, 7, 13.

Exercises to be handed in

Exercise 1. Do **[DF]**, §4.4, exercise 8.

Exercise 2. Do **[DF]**, §4.4, exercise 18.

Exercise 3. Prove Cauchy's Theorem using the Sylow theorems. Cauchy's theorem states: *If G is a finite group and p is a prime number dividing the group order $|G|$ then G has an element of order p.*

Exercise 4. Let $q = p^\alpha$ where α is a positive integer and p is a prime number. Let $\mathbb{F}_q$ be a field with q-elements. (We will prove later that such a field always exists and that it is unique up to isomorphism. If $q = p$, then $\mathbb{F}_p$ is just $\mathbb{Z}/p\mathbb{Z}$.)

(1) Let $\mathrm{GL}_n(\mathbb{F}_q)$ be the set of invertible $n \times n$ matrices with entries in $\mathbb{F}_q$. What is the group order $|\mathrm{GL}_n(\mathbb{F}_q)|$? (This is a standard problem, you may have seen it before. *Hint:* Let $v_1, v_2, \ldots, v_n$ be the columns of a matrix in $\mathrm{GL}_n(\mathbb{F}_q)$. What are the possibilities for v_1? Given v_1 what are the possibilities for v_2, and so forth.)

(2) Let $B \leq \mathrm{GL}_n(\mathbb{F}_q)$ be the subgroup of all upper triangular matrices with 1's on the diagonal. Prove that B is a p-sylow subgroup.

Exercise 5. Do **[DF]**, §4.5, Exercise 18.

Hard Exercises (optional, for extra credit)

Exercise 6. **[DF]**, §4.5, 16.

Exercise 7. **[DF]**, §4.5, 29.

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http://www.math.lsa.umich.edu/~hderksen/math594.w06/index.html