MATH 594, WINTER 2006, PROBLEM SET 4

DUE: WEDNESDAY, 2/15/2006

Warm-up (not to be handed in)

[DF], §5.4, Exercise 15, §5.5, Exercises 11, 22, 23, §6.1, Exercises 1, 7 (ignore infinite groups), 17, 31.

1. Exercises to be handed in

Exercise 1. Do [DF], §5.4, Exercise 18.

Exercise 2. Do [DF], §5.5. Exercise 12.

Exercise 3. Suppose that \( \sigma \in S_n \) has cycle type
\[
(m_1^{k_1}, m_2^{k_2}, \ldots, m_r^{k_r}) = (m_1, m_1, \ldots, m_{k_1}, m_2, m_2, \ldots, m_{k_2}, \ldots, m_r, m_r, \ldots, m_{k_r})
\]
where \( m_1 > m_2 > \cdots > m_r \) are \( k_1, k_2, \ldots, k_r \) are positive integers. Let \( G = C_{S_n}(\sigma) \) be the centralizer group of \( \sigma \).

(a) Identify a normal subgroup \( H \cong Z_{m_1}^{k_1} \times \cdots \times Z_{m_r}^{k_r} \) of \( G \) and show that \( G/H \cong K := S_{k_1} \times S_{k_2} \times \cdots \times S_{k_r} \).

(b) Show that \( G \) is isomorphic to a semidirect product of \( H \) and \( K \). (In fact, \( G \) is a wreath product.)


Exercise 5. Do [DF], §6.1, Exercise 32.

Hard Exercises (optional, for extra credit)


Exercise 7. Do [DF], §5.5, Exercise 24. (You may use §4.5, Exercise 56).

Harm Derksen, 3067EH, 763 2309
Office hours: MWF 3-4pm.
http://www.math.lsa.umich.edu/~hderksen/math594.w06/index.html