MATH 594, WINTER 2006, PROBLEM SET 6

DUE: MONDAY, 3/20/2006

Warm-up (not to be handed in)

[DF], §13.3, exercise 1, §13.4, exercises 1, 2, 3, §13.5, exercises 1, 2, 7, 8, 9.

1. Exercises to be handed in


Exercise 2. Do [DF], §13.4, exercise 5.


Hard Exercises (optional, for extra credit)

Exercise 6. Let ζ be a 17-th root of unity. Define

\[ \alpha_1 = \zeta + \zeta^2 + \zeta^4 + \zeta^8 + \zeta^{-1} + \zeta^{-2} + \zeta^{-4} + \zeta^{-8}, \]
\[ \alpha_2 = \zeta + \zeta^4 + \zeta^{-1} + \zeta^{-4}, \]
and \( \alpha_3 = \zeta + \zeta^{-1} \). Prove that \( \mathbb{Q}(\zeta)/\mathbb{Q}(\alpha_3), \mathbb{Q}(\alpha_3)/\mathbb{Q}(\alpha_2), \mathbb{Q}(\alpha_2)/\mathbb{Q}(\alpha_1), \mathbb{Q}(\alpha_1)/\mathbb{Q} \)
are all quadratic field extensions. Conclude that it is possible to construct a
regular 17-gon with compass and straightedge.

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http://www.math.lsa.umich.edu/~hderksen/math594.w06/index.html