1. Find the multiplicity of the ring $K[[x^7, x^{11}, x^{13}]] \subseteq K[[x]]$, where $K$ is a field and $x$ is a formal power series indeterminate.

2. Let $Y, X_1, \ldots, X_n, \ldots$ be countably many indeterminates over a field $K$. Let $R = K[Y, X_1, \ldots, X_n, \ldots]$. Let $I = (X_n Y^n : n \geq 1) R$. Let $J = (X_1, \ldots, X_n, \ldots) R$. Let $S = R_Y$, which is $R$-flat. Show that $IS :S JS = S$, while $(I :_R J)S = IS$. (When $J$ is finitely generated, colon does commute with flat change of rings.)

3. Let $M$ be a finitely generated module of dimension $d$ over a local ring $R$ of dimension $d$. Let $x_1, \ldots, x_d$ be a system of parameters for $R$, and let $I = (x_1, \ldots, x_d) R$. Let $n_1, \ldots, n_d$ be given positive integers, let $y_i = x_n^i$, $1 \leq i \leq d$, and let $J = (y_1, \ldots, y_d) R$. Is it necessarily true that $e_J(M) = (n_1 \cdots n_d) e_I(M)$? Prove your answer.

4. Let $T = K[X, Y, Z, U, V, W]$, and let $S = (UX + Y^2 + Z^2) = K[x, y, z, u, v, w]$. Let $m = (x, y, z, u, v, w) S$, and let $P = (x, y, z) S$, which is prime. Show that $P^{(2)} \nsubseteq m^2$.

5. (a) Let $(A, P) \subseteq (R, m)$ be an integral extension of quasilocal domains. Let $t$ be an indeterminate, and let $F \in R[t]$ be a polynomial at least one of whose coefficients is a unit. Show that $F$ has a multiple in $A[t]$ at least one of whose coefficients is a unit.

(b) Show that if $(R, m, K)$ is a complete local domain, then the completion of $R(t) = R[t]_{m R[t]}$, where $t$ is an indeterminate, is of pure dimension. (You may assume that $R$ is module-finite over $A$ regular. Then $R \to A^{\oplus h}$ for some $h$, and $R \otimes_A A(t) \to A(t)^{\oplus h}$. Show that $R \otimes_A A(t) \cong R(t)$ using (a), and also use that $A(t)$ is regular.)

6. Let $r, s \geq 1$ be integers and let $X_1, \ldots, X_r, Y_1, \ldots, Y_s$ be indeterminates over the field $K$. Let $S = K[X_i Y_j : 1 \leq i \leq r, 1 \leq j \leq s] \subseteq K[X_1, \ldots, X_r, Y_1, \ldots, Y_s]$, and let $R$ be the localization of $S$ at the maximal ideal generated by all the $X_i Y_j$. What is the multiplicity of $R$?

**BONUS** Let $0 < a_1 < \cdots < a_k$ be integers whose greatest common divisor is 1. (They need not be relatively prime in pairs.) Generalize problem 1. by finding the multiplicity of the ring $K[[t^{a_1}, \ldots, t^{a_k}]] \subseteq K[[t]]$.