MATH 417 9/16/2015

Note Title 9/16/2015

Quiz: Find all solutions to the system of linear equations in unknowns x, y, z, t, u

$$\begin{aligned} 3z + t + u &= 5 \\ x + y + z \quad + u &= 4 \\ 2x + 2y + 5z \quad + u &= 6. \end{aligned}$$

For those who are done:

Challenge (not graded): Consider the set $\mathbb{R}_+$ of all positive real numbers with operations: $\oplus$ $\quad x \oplus y = x \cdot y$

$\odot \quad \lambda \odot x = x^{\lambda}, \ \lambda \in \mathbb{R}.$

Is this a vector space?

If not, <u>why</u>? (What goes wrong?) If yes, <u>why</u>? What is "$\textcircled{0}$"?

$$\log_{10} : \mathbb{R}_+ \longrightarrow \mathbb{R} \qquad\qquad \mathbb{R} \xrightarrow{10^x} \mathbb{R}_+$$

$$\left.\begin{aligned} \log_{10}(x \cdot y) &= \log_{10}(x) + \log_{10}(y) \\ \log_{10}(x^{\lambda}) &= \lambda \log_{10}(x) \\ 1 = \textcircled{0} \qquad \log_{10}(\textcircled{0}) &= 0 \end{aligned}\right\} \ \mathbb{R} \text{ is a vector space (over } \mathbb{R})$$

Vector spaces in the book are Chapter 4
(called <u>linear spaces</u>)

A vector space (= linear space) is a set V
with an operation "+" $(x+y)$ ← addition
and, for every real number λ, $\lambda \cdot ?$ (λx) ← scalar multi-plication
which satisfy commutativity, associativity, 0, inverse $(-?)$
for +,

$1 \cdot x = x$, $\lambda \cdot (\mu \cdot x) = (\lambda \cdot \mu) \cdot x$,
distributivities.

- $m \times n$ matrices $+$, scalar multiplication, form a vector space. (Special cases: $m \times 1$ column vectors, $1 \times n$ row vectors)

- solutions of a homogeneous system of linear equations (right hand side is 0).

- Calculus example: Real functions of a real variable with given domain D.

$$(f+g)(x) = f(x) + g(x)$$

$$(\underset{\sim}{\lambda} \cdot f)(x) = \lambda(f(x))$$

scalar multiple of the function f, applied to $x \in D$

a product of two real numbers

$\cup$

- Solutions y of a given homogeneous linear differential equation

$$y^{(n)} + a_{n-1}(x)\, y^{(n-1)} + \cdots + a_1(x)\, y' + a_0(x)\, y = 0$$

A vector subspace of a vector space V

is a subset $W \subseteq V$ which is closed

under the vector space operations which means:

a linear combination of elements of W

is always in W.

It suffices to verify that • for $x, y \in W$

we always have $x+y \in W$

• for $x \in W$, $\lambda \in \mathbb{R}$

$\lambda x \in W$.

<u>Calculus 3 examples</u>: $\mathbb{R}^3 = \left\{ \begin{pmatrix} x \\ y \\ z \end{pmatrix} \middle| \, x, y, z \in \mathbb{R} \right\}$

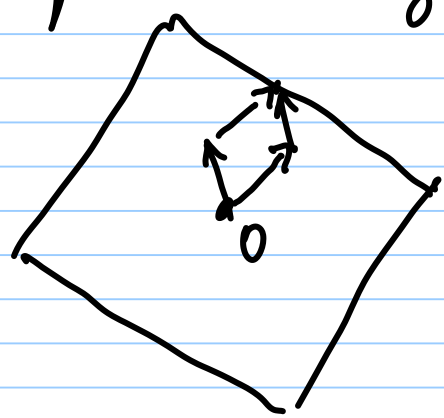
the set of all

such that

$\mathbb{R}^3$ is a vector space. What are its vector subspaces?

① $\mathbb{R}^3$ itself.

② Any plane through the origin.



$0 = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$

③ Any line through the origin

④ $\{0\} = \left\{ \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} \right\}$

not a subspace

$S = \{0, 1, -1\}$

$1 + (-1) = 0$

$1 + 1 \notin S$

$\frac{1}{2} \cdot 1 \notin S$

HW ① Consider the set of all polynomials with real coefficient of degree ≤ 5

$$a_5 x^5 + a_4 x^4 + a_3 x^3 + a_2 x^2 + a_1 x + a_0$$

$a_0, \dots, a_5 \in \mathbb{R}$

ⓐ Is this a vector space with the operations $+$, multiplying by a number?

ⓑ Is it a vector subspace of one of the vector spaces discussed in class today?

② (1.2.32 in 5th edition): Find a polynomial of degree 3 ($a + bx + cx^2 + hx^3$ $h \neq 0$) whose graph passes through the points

$(0,1)$; $(1,0)$; $(-1,0)$; $(2,-15)$.

x y