Note Title 9/23/2015

MATH 417 9/23/2015

<u>Quiz</u>: Write down the matrix of the following row operation on 4×4 matrices:

"Add 6 times the second row to the 4th row." $\circledast$

<u>Challenge</u> (not graded): What is the matrix of the composite operation of doing $\circledast$ first and then adding the 4th row to the third row?

Solution to quiz:

$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 6 & 0 & 1 \end{pmatrix}$$

Solution to challenge:

$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 6 & 0 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 6 & 1 & 1 \\ 0 & 6 & 0 & 1 \end{pmatrix}$$

Another method: just do the op's on I:

$$\begin{pmatrix} 1 & & & \\ & 1 & & \\ & & 1 & \\ & & & 1 \end{pmatrix} \rightsquigarrow \begin{pmatrix} 1 & & & \\ & 1 & & \\ & & 1 & \\ & 6 & & 1 \end{pmatrix} \rightsquigarrow \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 6 & 1 & 1 \\ 0 & 6 & 0 & 1 \end{pmatrix}$$

<u>The rank of a matrix</u> is the number of pivot in its RREF

<u>Example</u>: Find rank(A) where

$$A = \begin{pmatrix} 2 & 1 & 0 & 3 \\ 1 & 1 & 1 & 2 \\ 4 & 3 & 2 & 7 \end{pmatrix}$$

<u>Solution</u>: $\begin{pmatrix} 2 & 1 & 0 & 3 \\ 1 & 1 & 1 & 2 \\ 4 & 3 & 2 & 7 \end{pmatrix} \sim \begin{pmatrix} 1 & 1 & 1 & 2 \\ 2 & 1 & 0 & 3 \\ 4 & 3 & 2 & 7 \end{pmatrix} \begin{matrix} -2 \\ \\ -4 \end{matrix}$

$$\sim \begin{pmatrix} 1 & 1 & 1 & 2 \\ 0 & -1 & -2 & -1 \\ 0 & -1 & -2 & -1 \end{pmatrix} \sim \begin{pmatrix} 1 & 1 & 1 & 2 \\ 0 & -1 & -2 & -1 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

I know that $\operatorname{rank}(A) = 2$.

A matrix A with m rows and n columns, $m \leq n$ has a right inverse B if and only if $\operatorname{rank} A = m$

$$m \underbrace{\boxed{\quad A \quad}}_{n} \boxed{B} = I_m$$

$$(A \mid I) \sim \text{RREF}.$$

Example: Find $\underline{\underline{a}}$ right inverse, if any, of

$$\begin{pmatrix} 2 & 3 & 1 \\ 1 & 2 & 2 \end{pmatrix}$$

Solution:

$$\left(\begin{array}{ccc|cc} 2 & 3 & 1 & 1 & 0 \\ 1 & 2 & 2 & 0 & 1 \end{array}\right) \sim \left(\begin{array}{ccc|cc} 1 & 2 & 2 & 0 & 1 \\ 2 & 3 & 1 & 1 & 0 \end{array}\right) \overset{-2}{\sim} \left(\begin{array}{ccc|cc} 1 & 2 & 2 & 0 & 1 \\ 0 & -1 & -3 & 1 & -2 \end{array}\right)_{2}$$

$$\hookrightarrow \left(\begin{array}{ccc|cc} 1 & 2 & 2 & 0 & 1 \\ 0 & 1 & 3 & -1 & 2 \end{array}\right) \sim \left(\begin{array}{ccc|cc} 1 & 0 & -4 & 2 & -3 \\ 0 & 1 & 3 & -1 & 2 \end{array}\right) \qquad \begin{pmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \\ b_{31} & b_{32} \end{pmatrix}$$

$$\left(\begin{array}{ccc|c} 1 & 0 & -4 & 2 \\ 0 & 1 & 3 & -1 \end{array}\right)$$

t

$b_{11} = 2 + 4t$
$b_{21} = -1 - 3t$
$b_{31} = t$

$$\left(\begin{array}{ccc|c} 1 & 0 & -4 & -3 \\ 0 & 1 & 3 & 2 \end{array}\right)$$

s

$b_{12} = -3 + 4s$
$b_{22} = 2 - 3s$
$b_{32} = s$

The most general right inverse:

$$\begin{pmatrix} 2+4t & -3+4s \\ -1-3t & 2-3s \\ t & s \end{pmatrix}$$

$$\begin{pmatrix} 2 & 3 & 1 \\ 1 & 2 & 2 \end{pmatrix} \begin{pmatrix} 2+4t & -3+4s \\ -1-3t & 2-3s \\ t & s \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \quad \checkmark$$

The way the problem was phrased, we can set $s = t = 0$:

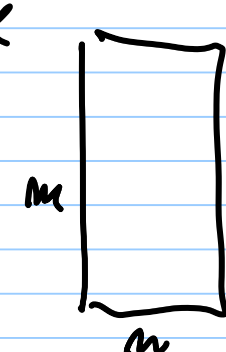
$$\begin{pmatrix} 2 & -3 \\ -1 & 2 \\ 0 & 0 \end{pmatrix} \begin{array}{l} \leftarrow \\ \leftarrow \\ \end{array}$$

$$(A \mid I) \sim (\mathrm{RREF} \mid B)$$

An answer: B in the rows

corresponding to pivot columns. (other rows 0).
non-pivot

If A is an $m \times n$ matrix with $m \geq n$ then A has a left inverse if and only if $\operatorname{rank} A = n$



To find a left inverse: Find a right inverse B of A^T, transpose it:

Answer: B^T

Example: Find a left inverse, if any, of

$$A = \begin{pmatrix} 1 & 1 \\ 2 & 2 \\ 1 & 2 \end{pmatrix}$$

Solution: A^T

$$\left(\begin{array}{ccc|cc} 1 & 2 & 1 & 1 & 0 \\ 1 & 2 & 2 & 0 & 1 \end{array}\right) \sim \left(\begin{array}{ccc|cc} 1 & 2 & 1 & 1 & 0 \\ 0 & 0 & 1 & -1 & 1 \end{array}\right) \quad -1$$

2

$$\left(\begin{array}{ccc|cc} 1 & 2 & 0 & 2 & -1 \\ 0 & 0 & 1 & -1 & 1 \end{array}\right)$$

$$\begin{pmatrix} 1 & 2 & 1 \\ 1 & 2 & 2 \end{pmatrix}\begin{pmatrix} 2 & -1 \\ 0 & 0 \\ -1 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \checkmark$$

$$\Big\downarrow T$$

Answer: $\begin{pmatrix} 2 & 0 & -1 \\ -1 & 0 & 1 \end{pmatrix}$

$$\begin{pmatrix} 2 & 0 & -1 \\ -1 & 0 & 1 \end{pmatrix}\begin{pmatrix} 1 & 1 \\ 2 & 2 \\ 1 & 2 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

HW:

Due Monday 9/28

1. Find the matrix of a composite operation on a 3×3 matrix:

"First switch the first and second row. Then add twice the 3rd row to the first row."

2. Find the rank of the matrix

$$\begin{pmatrix} 1 & 1 & 2 & 1 \\ 1 & 0 & 1 & 1 \\ 2 & 1 & 3 & 2 \\ 3 & 1 & 4 & 3 \end{pmatrix}$$

③ Find a right inverse, if any, of

$$\begin{pmatrix} 1 & -1 & 1 & 1 \\ 2 & -2 & 1 & 3 \end{pmatrix}$$

④ Find a left inverse, if any, of

$$\begin{pmatrix} 1 & 1 \\ 2 & 3 \\ 1 & -2 \end{pmatrix}.$$