MATH 417 9/25/2015

Note Title 9/25/2015

<u>Midterm 1</u> : Wed 9/30 : in class 1:10 - 2:00

5 questions (maybe some (a), (b) parts)

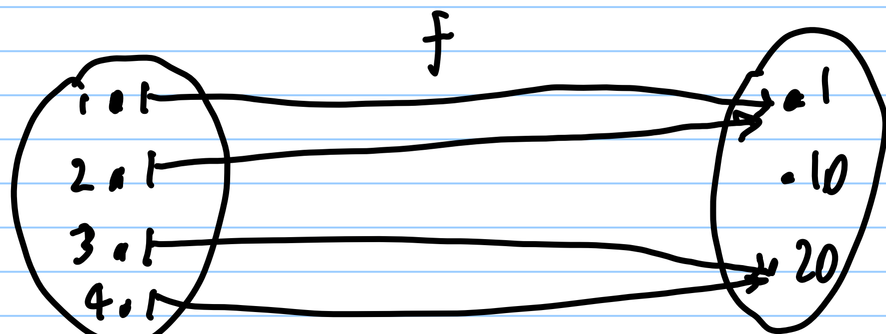
① solving systems of linear equations
(the most general case, including the possibility
of no solution or ∞ many solutions
expressed by parameters)

2. Reduced row echelon form, Gauss elimination
3. Row operations and their matrices
4. Rank of a matrix
5. inverse, left inverse, right inverse.

Midterm policy: No calculators
(No notes or open books)

Linear transformations (homomorphisms of vector spaces)

First, let us talk about mappings between sets.



$f(1)=1$

$f(2)=1$

$f(3)=20$

$f(4)=20$

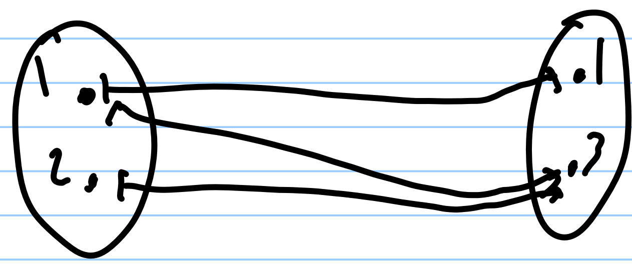
domain
source $\rightarrow S=\{1,2,3,4\}$

$T=\{1,10,20\} \leftarrow$ codomain
target

A mapping (map) $f: S \rightarrow T$ assigns to every element of S precisely one element of T.

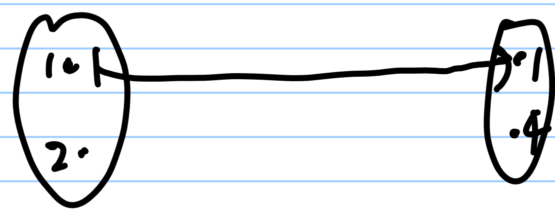
Not a mapping:

multiply defined



$$\left.\begin{array}{l} f(1) = 1 \\ f(1) = 3 \end{array}\right\} \times$$

$$f(2) = 3$$



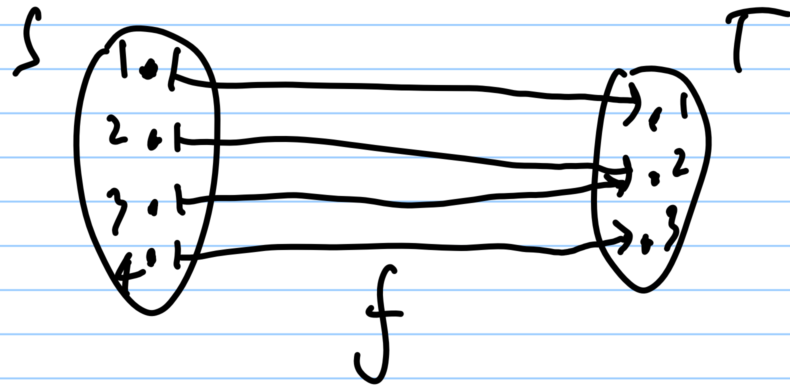
$$f(1) = 1$$

$f(2)$ undefined $\times$

Onto mapping

↑

surjective



$f(1) = 1$

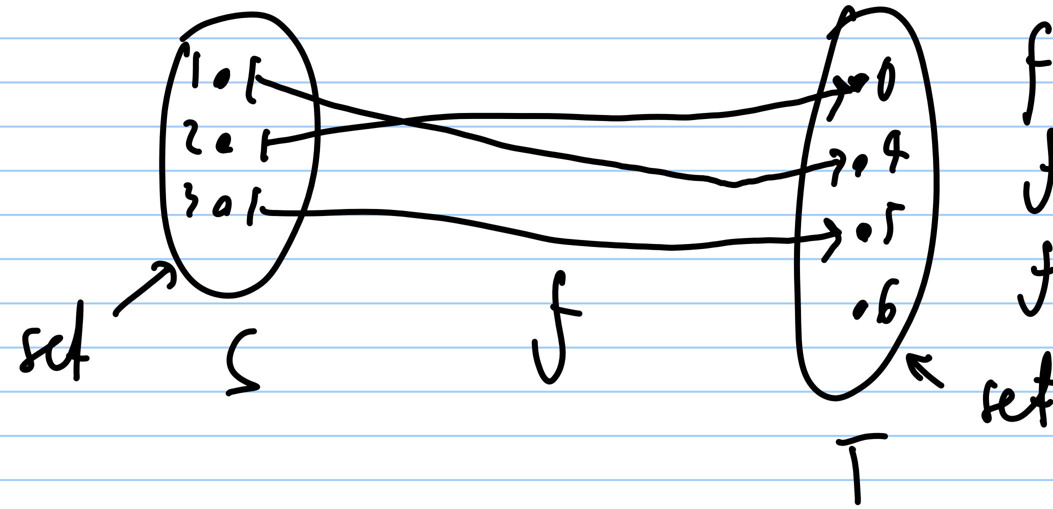
$f(2) = f(3) = 2$

$f(4) = 3$

For every $x \in T$ there is a $t \in S$ with $f(t) = x$.

1-1 mapping

injective



$f(1) = 4$

$f(2) = 0$

$f(3) = 5$

We cannot have two different elements $x, y \in S$ such that $f(x) = f(y)$

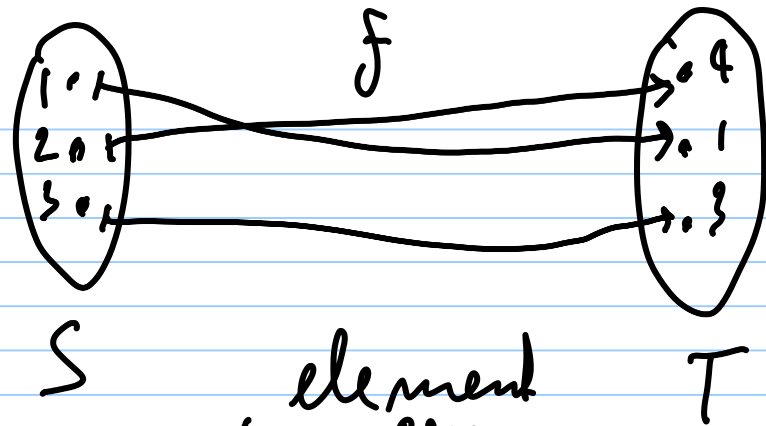
$x \neq y$

In set theory, listing an element of a set multiple times makes the same set (also, order, if any, does not matter)

$\{1, 2\} = \{2, 1\} = \{1, 2, 1\} = \{2, 2, 1\} = \cdots$

"a tangent"

A mapping is called <u>bijective</u> if it is both onto and 1-1.



$f(1) = 1$

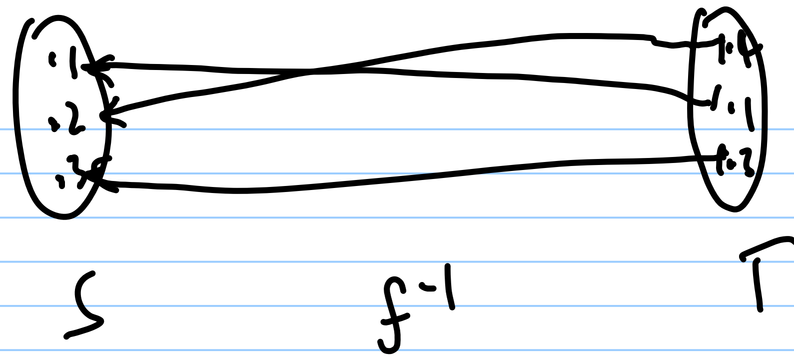
$f(2) = 4$

$f(3) = 3$

element sign

For every $t \in T$ there is precisely one $s \in S$ such that $f(s) = t$.

A bijective mapping has an inverse



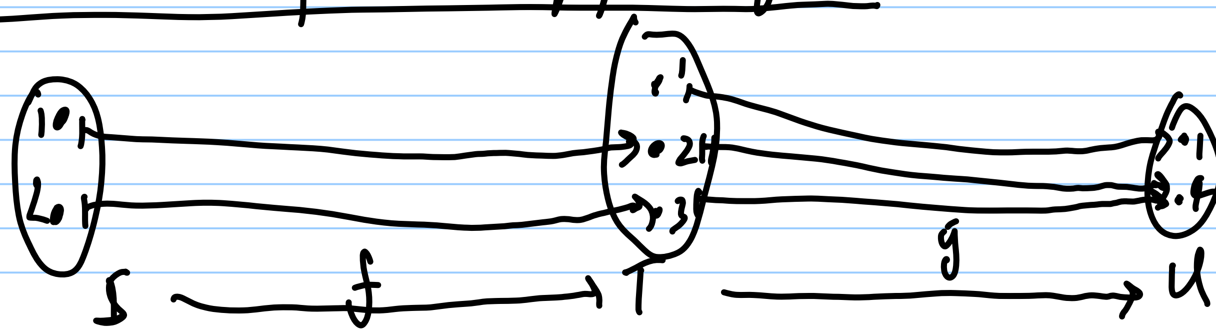
$f^{-1}(1) = 1$

$f^{-1}(4) = 2$

$f^{-1}(3) = 3$

(reverse the direction of $\longmapsto$).

<u>Composition of mappings:</u>



$f(1) = 2$

$f(2) = 3$

$g(1) = 1$

$g(2) = 4$

$g(3) = 4$

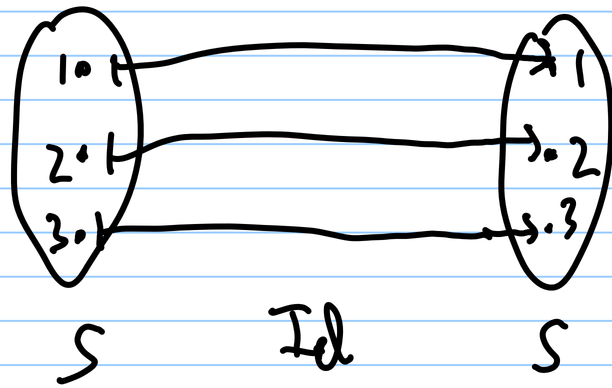
composition

$$g \circ f(x) = g(f(x))$$

$$g \circ f(1) = g(f(1)) = g(2) = 4$$

$$g \circ f(2) = g(f(2)) = g(3) = 4$$

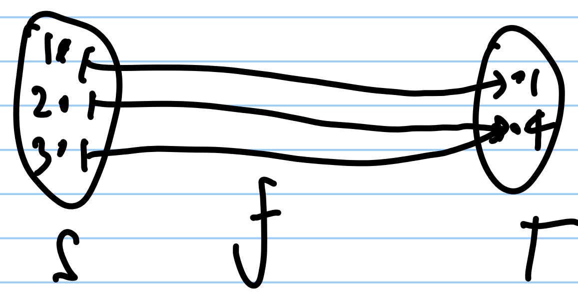
The identity mapping $Id_S : S \longrightarrow S$



$Id_S(x) = x$

The inverse mapping $f^{-1} : T \to S$ of a bijective mapping $f : S \to T$ has:

$$f^{-1} \circ f = Id_S \quad , \quad f \circ f^{-1} = Id_T .$$



onto

$f(1) = 1$

$f(2) = 4 = f(3)$

A right inverse satisfies $f \circ g = Id_T$

$g(1) = 1$

$g(4) = 2$

another solution:

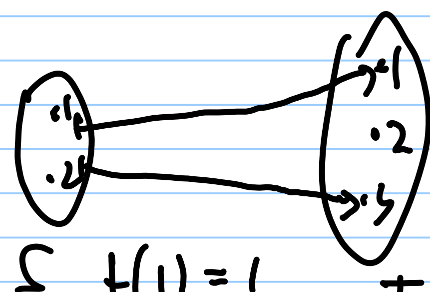
$\tilde{g}(1) = 1$

$\tilde{g}(4) = 3$

$f \circ \tilde{g} = Id_T$

An injective mapping $f : S \to T$ can have a left inverse $g : T \to S$

$g \circ f = Id_S$



$f(1) = 1$

$f(2) = 3$

$g(1) = 1$
$g(3) = 2$
$g(2) = 1$

$g \circ f = Id_S$

$\tilde{g}(1) = 1$
$\tilde{g}(3) = 2$
$\tilde{g}(2) = 2$

$\tilde{g} \circ f = Id_S$.

A mapping $f : V \to W$ where V, W are vector spaces is called a <u>linear transformation</u> (or homomorphisms of vector spaces) when $\quad f(x+y) = f(x) + f(y) \quad$ for all $x, y \in V$

$\quad f(\lambda x) = \lambda f(x) \quad \lambda \in \mathbb{R}.$

real numbers

HW Due Monday 9/28.

Is the following mapping

$\underbrace{\text{injective}}_{1-1}$, $\underbrace{\text{surjective}}_{\text{onto}}$, bijective ?

If it is bijective, give an inverse
If it is injective (resp. surjective), give a left (resp. right) inverse.

① $f_1 : \mathbb{R} \longrightarrow \mathbb{R}$

$f_1(x) = x^2$

② $f_2 : \overbrace{[0, \infty)}^{0 \leq x < \infty} \longrightarrow \mathbb{R}$

$f_2(x) = x^2$

③ $f_3 : \mathbb{R} \longrightarrow [0, \infty)$

$$f_3(x) = x^2$$

④ $f_4 : [0, \infty) \longrightarrow [0, \infty)$

$$f_4(x) = x^2.$$