MATH 417 10/7/2015

Note Title 10/7/2015

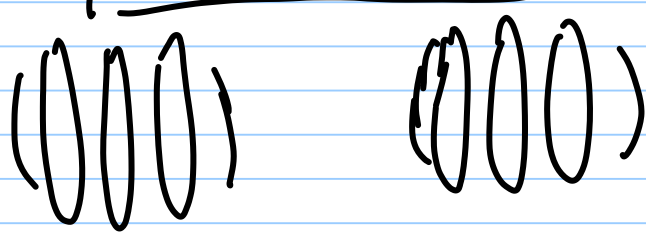
Quiz: Do the following vectors span $\mathbb{R}^3$?

$$\begin{pmatrix}1\\-1\\2\end{pmatrix}, \begin{pmatrix}1\\1\\1\end{pmatrix}, \begin{pmatrix}2\\0\\3\end{pmatrix}, \begin{pmatrix}4\\2\\5\end{pmatrix}, \begin{pmatrix}6\\2\\7\end{pmatrix}.$$

Challenge (not graded): Let A_{BC} be the base change matrix between the bases made of the columns of a matrix B and the columns of a matrix C. What is the relationship between A_{BC}, B and C?

The columns of A_{BC} are coefficients of linear combinations of vectors of C which give vectors of B.

$$\boxed{B = C A_{BC}}$$



Example: Find the base change matrix from

$B: \begin{pmatrix} 1 & 1 & 2 \\ 1 & 2 & 3 \\ 0 & 1 & 2 \end{pmatrix}$ to $C: \begin{pmatrix} 1 & 1 & 0 \\ 2 & 1 & 1 \\ 1 & 2 & -2 \end{pmatrix}$

$$C\underbrace{A_{BC}}_{?} = B \qquad\qquad (C \mid B) \sim (I \mid A_{BC})$$

$$\left(\begin{array}{ccc|ccc} 1 & 1 & 0 & 1 & 1 & 2 \\ 2 & 1 & 1 & 1 & 2 & 3 \\ 1 & 2 & -2 & 0 & 1 & 2 \end{array}\right) \sim \left(\begin{array}{ccc|ccc} 1 & 1 & 0 & 1 & 1 & 2 \\ 0 & -1 & 1 & -1 & 0 & -1 \\ 0 & 1 & -2 & -1 & 0 & 0 \end{array}\right)$$

$$2$$

$$\left(\begin{array}{ccc|ccc} 1 & 0 & 1 & 0 & 1 & 1 \\ 0 & 1 & -1 & 1 & 0 & 1 \\ 0 & 0 & -1 & -2 & 0 & -1 \end{array}\right) \sim \left(\begin{array}{ccc|ccc} 1 & 1 & 0 & 1 & 1 & 2 \\ 0 & 1 & -1 & 1 & 0 & 1 \\ 0 & 1 & -2 & -1 & 0 & 0 \end{array}\right)$$

$$5$$

$$\left(\begin{array}{ccc|ccc} 1 & 0 & 0 & -2 & 1 & 0 \\ 0 & 1 & 0 & 3 & 0 & 2 \\ 0 & 0 & 1 & 2 & 0 & 1 \end{array}\right)$$

$$B: \begin{pmatrix} 1 & 1 & 2 \\ 1 & 2 & 3 \\ 0 & 1 & 2 \end{pmatrix} \quad \text{to} \quad C: \begin{pmatrix} 1 & 1 & 0 \\ 2 & 1 & 1 \\ 1 & 2 & -2 \end{pmatrix}$$

$$A_{BC} = \begin{pmatrix} -2 & 1 & 0 \\ 3 & 0 & 2 \\ 2 & 0 & 1 \end{pmatrix}$$

$$\begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} = -2\begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix} + 3\begin{pmatrix} 1 \\ 1 \\ 2 \end{pmatrix} + 2\begin{pmatrix} 0 \\ 1 \\ -2 \end{pmatrix}$$

$$\begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix} = 1\begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix} + 0\begin{pmatrix} 1 \\ 1 \\ 2 \end{pmatrix} + 0\begin{pmatrix} 0 \\ 1 \\ -2 \end{pmatrix}$$

$$\begin{pmatrix} 2 \\ 3 \\ 2 \end{pmatrix} = 0\begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix} + 2\begin{pmatrix} 1 \\ 1 \\ 2 \end{pmatrix} + 1\begin{pmatrix} 0 \\ 1 \\ -2 \end{pmatrix}$$

✓

We are really interested in subspaces of some $\mathbb{R}^N$

If $v_1, \dots, v_m$ are linearly independent vectors in $\mathbb{R}^N$ (column vectors $N \times 1$) then we can take the subspace $\langle v_1, \dots v_m \rangle \subseteq \mathbb{R}^N$ which is the set of all linear combinations of $v_1, \dots, v_m$.

B: $v_1, \dots v_m$, if they are linearly independent, $N \geq m$

form a basis of $\langle v_1, \dots, v_m \rangle$. We can ask if

C: $w_1, \dots w_m \in \mathbb{R}^N$ are some other vectors, do they span the same subspace? We can solve this by trying to find the base change matrix A_{BC}.

Example: Are the vectors

$$B: \begin{pmatrix} 1 \\ 2 \\ 1 \\ 4 \end{pmatrix}, \begin{pmatrix} 1 \\ 1 \\ 2 \\ 3 \end{pmatrix}, \begin{pmatrix} 2 \\ 3 \\ 3 \\ 8 \end{pmatrix}$$

a basis of the same subspace of $\mathbb{R}^4$ as

$$C: \begin{pmatrix} 3 \\ 5 \\ 4 \\ 12 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \\ -1 \\ 1 \end{pmatrix}, \begin{pmatrix} -1 \\ 0 \\ -3 \\ -2 \end{pmatrix}$$

$(C \mid B)$

$$\begin{array}{c} \\ \\ -1 \\ -3 \end{array} \left(\begin{array}{ccc|ccc} 3 & 0 & -1 & 1 & 1 & 2 \\ 5 & 1 & 0 & 2 & 1 & 3 \\ 4 & -1 & -3 & 1 & 2 & 3 \\ 12 & 1 & -2 & 4 & 3 & 8 \end{array}\right) \sim \left(\begin{array}{ccc|ccc} 3 & 0 & -1 & 1 & 1 & 2 \\ 1 & 2 & 3 & 1 & -1 & 0 \\ 4 & -1 & -3 & 1 & 2 & 3 \\ 0 & 4 & 7 & 1 & -3 & -1 \end{array}\right)$$

$$\left(\begin{array}{ccc|ccc} 1 & 2 & 3 & 1 & -1 & 0 \\ 3 & 0 & -1 & 1 & 1 & 2 \\ 4 & -1 & -3 & 1 & 2 & 3 \\ 0 & 4 & 7 & 1 & -3 & -1 \end{array}\right) \sim \left(\begin{array}{ccc|ccc} 1 & 2 & 3 & 1 & -1 & 0 \\ 0 & -6 & -10 & -2 & 4 & 2 \\ 0 & -9 & -15 & -3 & 6 & 3 \\ 0 & 4 & 7 & 1 & -3 & -1 \end{array}\right) -3/2$$

$$\left(\begin{array}{ccc|ccc} 1 & 2 & 3 & 1 & -1 & 0 \\ 0 & -3 & -5 & -1 & 2 & 1 \\ 0 & 4 & 7 & 1 & -3 & -1 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{array}\right) \sim \left(\begin{array}{ccc|ccc} 1 & 2 & 3 & 1 & -1 & 0 \\ 0 & -3 & -5 & -1 & 2 & 1 \\ 0 & 1 & 2 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{array}\right)$$

$$\left(\begin{array}{ccc|ccc} 1 & 2 & 3 & 1 & -1 & 0 \\ 0 & 1 & 2 & 0 & -1 & 0 \\ 0 & -3 & -5 & -1 & 2 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{array}\right) \sim \left(\begin{array}{ccc|ccc} 1 & 0 & -1 & 1 & 1 & 0 \\ 0 & 1 & 2 & 0 & -1 & 0 \\ 0 & 0 & 1 & -1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{array}\right) \sim \left(\begin{array}{ccc|ccc} 1 & & & 0 & 2 & 1 \\ & 1 & & 2 & -3 & -2 \\ & & 1 & -1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{array}\right)$$

<u>Yes</u>

$$A_{BC} = \begin{pmatrix} 0 & 2 & 1 \\ 2 & -3 & -2 \\ -1 & 1 & 1 \end{pmatrix} \quad B: \begin{pmatrix} 1 \\ 2 \\ 1 \\ 4 \end{pmatrix}, \begin{pmatrix} 1 \\ 1 \\ 2 \\ 3 \end{pmatrix}, \begin{pmatrix} 2 \\ 3 \\ 3 \\ 8 \end{pmatrix}$$

a basis of the same subspace of $\mathbb{R}^4$ as

$$C: \begin{pmatrix} 3 \\ 5 \\ 4 \\ 12 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \\ -1 \\ 1 \end{pmatrix}, \begin{pmatrix} -1 \\ 0 \\ -3 \\ -2 \end{pmatrix}$$

$$\begin{pmatrix}1\\2\\1\\4\end{pmatrix} = 0\cdot\begin{pmatrix}3\\5\\4\\12\end{pmatrix} + 2\begin{pmatrix}0\\1\\-1\\1\end{pmatrix} + (-1)\begin{pmatrix}-1\\0\\-3\\-2\end{pmatrix} \quad \text{etc.}$$

HW ① Find the base change matrix A_{BC} of $\mathbb{R}^3$ from

$$B: \begin{pmatrix}1\\1\\3\end{pmatrix}, \begin{pmatrix}1\\2\\1\end{pmatrix}, \begin{pmatrix}4\\5\\6\end{pmatrix}$$

to

$$C: \begin{pmatrix}1\\2\\2\end{pmatrix}, \begin{pmatrix}1\\1\\1\end{pmatrix}, \begin{pmatrix}2\\3\\2\end{pmatrix}.$$

② Do the following linearly independent sets of vectors span the same subspace of $\mathbb{R}^5$? If so, find the base change A_{BC}:

B: $\begin{pmatrix}1\\1\\0\\1\\2\end{pmatrix}, \begin{pmatrix}1\\2\\1\\1\\1\end{pmatrix}, \begin{pmatrix}4\\6\\2\\2\\4\end{pmatrix}$

C: $\begin{pmatrix}0\\1\\1\\0\\-1\end{pmatrix}, \begin{pmatrix}2\\3\\1\\2\\3\end{pmatrix}, \begin{pmatrix}5\\8\\3\\3\\5\end{pmatrix}.$