MATH 395 PROBLEMS 6

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Regular problems:

1. Solve the following differential equation:

$$y' = \frac{x+y+1}{x+1}.$$

2. Solve:

$$y' = \frac{3y^2 - x^2}{2xy}.$$

[Hint: can you express the right hand side as a function of $\frac{y}{x}$?]

3. (a) Prove that the function $f(x, y) = 3y^{2/3}$ is not locally Lipschitz in the second variable (i.e. does not satisfy the hypothesis of the existence and uniqueness theorem for solutions of ordinary differential equations).

(b) Can you find initial conditions in which the differential equation $y' = 3y^{2/3}$ has more than one solution? [Hint: when looking, consider the points in the neighbourhood of which f is not Lipschitz.]

4. Using variation of constants, solve:

$$y' = \frac{y}{x} + \ln(x).$$

Challenge problem:

5. *The pendulum.* The ideal pendulum is an infinitely small ball hanging on an infinitely thin string of length r attached on the other end to a point P. Thus, the pendulum will move forth and back on a trajectory which is an arc of a circle.

Now the ball has a certain mass m. If g is the gravitational acceleration, there is a vertical force gm pulling the ball downward. This force decomposes to two orthogonal parts: one, which is parallel to the string, is balanced by the strength of the string and has no effect on the motion of the pendulum. The other one, which is perpendicular to the string, when divided by the mass of the ball, is its acceleration, or the second derivative of the angle between the string and the vertical axis.

(a) Formulate a differential equation describing the motion of the pendulum, i.e. the dependency of the angle of the string with the vertical axis on time t. Solve this equation as far as you can (you should be able to separate variables).

(b) Approximating $\sin x$ by x for small values of $|x|$, solve the resulting equation completely, and find an explicit formula approximately describing the motion of the pendulum provided that the angle at time $t = 0$ is small. (use the initial condition that the speed is 0 at time $t = 0$).