MATH 395 PROBLEMS 7

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Regular problems:

1. For the following system of linear differential equations, find an equivalent system of first order linear differential equations:

$$y'' = 4x^2y' + (\sin(x))z' + xy + \tfrac{z}{x} + \ln(x)$$
$$z'' = (\tan(x))y' + (\arctan(x))z' + \tfrac{y}{1+x^2} + x^3z + e^x.$$

2. Using the Wronskian, prove that the functions $x+1, e^x$ cannot be solutions of the same homogeneous linear differential equation

$$y'' + a_1(x)y' + a_2(x)y = 0$$

for any continuous functions

$$a_1, a_2 : \mathbb{R} \to \mathbb{R}.$$

3. Solve:

$$y''' - 6y'' + 9y' - 4y = 0.$$

4. Solve:
(a) $y'' - 3y' + 2y = e^{3x}$,
(b) $y'' - 3y' + 2y = x$.

Challenge problem:

5. Prove that if $y_1, \ldots, y_n : \mathbb{R} \to \mathbb{R}$ are functions with n continuous derivatives, and such that their Wronskian is everywhere non-zero, then there exists a homogenous order n linear differential equation whose space of solutions has basis $y_1, \ldots, y_n$.