# MATH 395 PROBLEMS 8

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**Regular problems:**

**1.** Find all solutions of the linear differential equation

$$y'' - 6y' + 8y = x^3.$$

**2.** Find a basis of the space of solutions of

$$y''' = y.$$

**3.** Find a basis of the space of solutions of

$$\begin{aligned} y_1' &= 3y_1 - 2y_3 \\ y_2' &= 2y_1 + y_2 - 2y_3 \\ y_3' &= y_1 \end{aligned}$$

**4.** Find all solutions of

$$\begin{aligned} y_1' &= y_1 - 2y_2 + e^{3x} \\ y_2' &= -2y_1 + y_2 + e^{4x} \end{aligned}$$

**Challenge problem:**

**5.** Find all possible Wronskians of $n$-tuples of solutions of
(a) The homogeneous system $(L)$:

$$y_i' = \sum_{j=1}^{n} a_{ij}(x) y_i, \quad i = 1, \ldots, n$$

(b) The homogeneous system $(\tilde{L})$:

$$y^{(n)} + a_1(x) y^{(n-1)} + \cdots + a_n(x) y = 0.$$

[The Wronskian satisfies a first order differential equation. Find the equation.]