Orientation moves

\[(R^2 \times R^2)^3\]

Twisted double switch

Back out double switch

\[C, B W^{-1} B^{-1} B \left( R^2 \times R^2 \right)^3 B^{-1} O B^{-1} W B^2 \left( R^2 \times R^2 \right)^3\]

Move

\[\begin{array}{c}
\begin{bmatrix}
4 & 0 \\
0 & 4
\end{bmatrix}
\end{array} \rightarrow
\begin{array}{c}
\begin{bmatrix}
0 & 4 \\
4 & 0
\end{bmatrix}
\end{array}
\]

\[\begin{array}{c}
\begin{bmatrix}
7 & 0 \\
0 & 7
\end{bmatrix}
\end{array} \rightarrow
\begin{array}{c}
\begin{bmatrix}
7 & 0 \\
0 & 7
\end{bmatrix}
\end{array}
\]
HW: Design a move reversing the orientation of two adjacent edges on the same face (e.g., $\frac{U}{2}$ $\frac{R}{2}$)

Note: Turns out, it is impossible to reverse the orientation of just one edge. (That needs proof!)

$B^{-1} O B^{-1} W B^2$

This step would also work in (x)
Orientation moves on corners:

\[(R_y R^{-1} y^{-1})^3\]

commutator \([R_y, R]\)

I should keep track of:

\[
\begin{array}{ccc}
\text{WR}&\text{BY}\\
2\text{C}&2\text{O}\\
\end{array}
\]

turn 120°

Step move:

\[O^{-1} W O W^{-1}\]

\[Q := O^{-1} W O^2 W^{-1} (R_y R^{-1} y^{-1})^3 W O^2 W^{-1} (R_y R^{-1} y^{-1})^3\]
Theorem about orienting corners on the cube: The total change of orientation of all corners (if I haven't moved them around)

if \( 0 \in \mathbb{Z}_3 \) \((\leq 2/3)\).
Again, this needs proof.

Orienting three corners 120° in the same direction:

\[ QGQ^{-1} G^{-1} \]

\[
\begin{align*}
&G & Y \\
&Y & G
\end{align*}
\]

\[
\begin{align*}
&G & R \\
&Y & Y
\end{align*}
\]

\[
\begin{align*}
&W & G \\
&R & R
\end{align*}
\]
Conway

\[ R^2 G B^{-1} Y^2 B G^{-1} \]

Switches three edges.

 HW: Use the Conway move to design a 3-switch of edges on one face.

Using the time lift for a new concept:
look at the subgroup of Rubik group legal moves on the cube:

generated by $B$ and $C$:

$$\{ B^{k_1} C^{l_1} / \quad k_1, l_1 = 0, 1, 2, 3 \}$$

$$B^{k_1} C^{l_1} B^{k_2} C^{l_2} = B^{k_1+k_2} C^{l_1+l_2}$$

This is an example of the direct product.

Let $G_1, H$ be two groups. The direct product
$G \times H$

consist of all pairs $(g, h)$, $g \in G$, $h \in H$.

The group operation in $G \times H$ is

$$(g_1, h_1) \cdot (g_2, h_2) = (g_1 g_2, h_1 h_2).$$

Remember that we proved that every group of order 2 or 3 is cyclic ($\cong \mathbb{Z}/2\mathbb{Z}$ or isomorphic to $\mathbb{Z}/3\mathbb{Z}$). But it is false for 4.

$\mathbb{Z}/2 \times \mathbb{Z}/2 \neq \mathbb{Z}/4$
\[\begin{array}{cccc}
\text{4/2} & 00 & 01 & 10 & 11 \\
00 & 00 & 01 & 10 & 11 \\
01 & 01 & 00 & 11 & 10 \\
10 & 10 & 11 & 00 & 01 \\
11 & 11 & 10 & 01 & 00 \\
\end{array}\] 

additively, \( x + x = 0 \) \( \forall \ x \in \mathbb{Z}/2 \cdot \mathbb{Z} \).

But in \( \mathbb{Z}/4 \), additively, \( 1 + 1 = 2 = 0 \).