Negative binomial distribution vs. geometric distribution. \[ \text{especially in special case} \]
\[ r = 1 \]

Negative binomial = repeat independent Bernoulli trials until we reach \( r \) failures. \( X = \text{number of successes} \)

\[ P(X = k) = \binom{r+k-1}{k} \left(\frac{r}{k}\right) \left(\frac{1}{r}\right)^k \left(\frac{1-\frac{1}{r}}{r}\right)^r \]

\[ = \binom{r+k-1}{k} \frac{r}{k} \left(\frac{1}{r}\right)^{k-1} \left(\frac{1-\frac{1}{r}}{r}\right)^r \]
Geometric random: \( r = 1 \) independent Bernoulli trials with success probability \( p \).

How many tries does it take until success?

\[
P\{Y = k\} = (1-p)^{k-1} p
\]

\( k \geq 1 \)

Negative binomial random variable (with probability \( 1-p \), \( r = 1 \)) + 1

Banach's matchbox problem

Banach - famous for functional analysis
A person who smokes pipe carries two match boxes, one in the left pocket, one in the right pocket. Each match box initially contained N matches.

Each time the person light up, he or she reaches for a match in either box with equal probability.

When the person discovers the box he/she reached for is empty, what is the probability there will be exactly I matches in the other box left?

Solution: Suppose the left pocket was the one discovered empty. (The event we are measuring is last pocket)
Negative binomial:

"failure" = left pocket \( v = N + 1 \)

"success" = right pocket \( k = N - l \)

\[
P\{X = k\} = \binom{v - 1}{k} \left(-\frac{1}{2}\right)^k \left(\frac{1}{2}\right)^{v-k} =
\]

\[
= \frac{\binom{v+k-1}{k}}{\binom{2N}{k}} \left(\frac{1}{2}\right)^{N+1} = \frac{\binom{2N-l}{N-l}}{\binom{2N}{N}} \left(\frac{1}{2}\right)^{N+1}
\]

Same probability for last pocket = right \( \frac{1}{2N} \)

Answer: \( \left(\frac{2N-l}{N-l}\right) \frac{1}{2N} = \left(\frac{2N-l}{N}\right) \frac{1}{2N} \)
THE HYPERGEOMETRIC DISTRIBUTION

"Similar to binomial, without replacement."

**Binomial**: Suppose we have \( N \) balls, \( m \) white, \( N-m \) black.

Select a ball \( n \) times, each time know it back.

\( X \): How many white ones have we selected.

\[
P(X = x) = \binom{N}{x} \left( \frac{m}{N} \right)^x \left( \frac{N-m}{N} \right)^{n-x}
\]
Hypergeometric: Exactly the same except do not throw the balls back after they are selected.

\[ P \{ X = i \} = \frac{\binom{m}{i} \binom{N-m}{m-i}}{\binom{N}{m}} = \frac{\binom{m}{i} \binom{N-m}{m-i}}{\binom{N}{m}} \]

N balls m white
m chosen

\[ X = \# \text{ (white & chosen) } \]
\[ \bar{m} = 3 \text{ chosen } \]
\[ \# \]

Think of N balls lined up in a row
\[ N = 6 \quad m = 4 \]
\[ \text{ (0 0 # 0 0 0) } \]
Example: A purchaser buys a set of 10 components.

Test 3 at random.
accepts the lot if all pass.

Suppose 30% of the lot have 4 defective component.
70% have 1 defective component.

What percentage will be rejected?

Solution: \( A = \text{accepts} \)

Hypergeometric

\[
P(A \mid 4 \text{ defective}) = \frac{3}{10} + P(A \mid 1 \text{ defective}) \cdot \frac{7}{10} =
\]

\( N = 10, m = 3, m' = 4 \)

\( N = 10, m = 3, m' = 1 \)

\( i = 0 \)

\( i' = 0 \)
\[
\begin{align*}
&= \frac{\binom{4}{6} \binom{6}{3}}{\binom{10}{3}} \cdot \frac{3}{10} + \frac{\binom{1}{0} \cdot \binom{9}{3}}{\binom{10}{3}} \cdot \frac{7}{10} = \frac{59}{100} \\
\text{Answer:} & \quad 46\% \\
\end{align*}
\]

HW
1. 4.76 p. 178-179
2. 4.79 p. 179
3. 4.85 p. 179