The remaining lemma for the model structure on simplicial sets:

If a cofibration satisfies the left lifting property with respect to fibrations of Kan complexes, then it satisfies the LLP with respect to all fibrations (hence is a retract of an anodyne extension).

Tierney-joyal (homotopy theory of simplicial sets)
A Kan complex $X$ is a minimal complex if for any diagram

\[ \partial \Delta^n \times I \to \partial \Delta^n \]

we have

\[ h_0 = h_1 : \Delta^n \to X. \]

(Theorem 12, p. 26 and n)

Theorem: If $X$ is a Kan complex then $X$ has
a strong deformation retract, which is minimal. □

Theorem: A homotopy equivalence of minimal \( \mathsf{H} \) hom complexes is an isomorphism. □

Similarly, we can define a minimal filtration on a filtration \( p: E \to X \) such that for any diagram

\[
\begin{array}{ccc}
\Delta^n \times I & \xrightarrow{\mu_n} & \Delta^n \\
\downarrow & & \downarrow a \\
\Delta^n \times I & \xrightarrow{h} & E
\end{array}
\]

filtration - homotopic implies.
we have \( h_0 = h_1 : \Delta_n \rightarrow E \).

**Theorem:** Every Kan fibration has a strong deformation retract \((\text{Id}_n \times X)\) which is a minimal fibration. \(\square\)

**Theorem:** Every minimal fibration is a bundle. \(\square\)
Definition: \( p : E \to X \) is called a bundle with fiber \( F \) if for any simplex \( \Delta_n \to X \) we have a diagram:

\[
\Delta_n \times F \xrightarrow{\times} \Delta_n \times X \overset{\pi_1}{\underset{\pi_2}{\twoheadrightarrow}} \Delta_n
\]

The key point of Lemma \( \Theta \) is that bundles are classified by maps into \( \mathbb{B} \text{Aut}(F) \).
\[ E \longrightarrow B(\text{Aut}(F), \text{Aut}(F) \times) \xrightarrow{\text{simply connected}} \]

\[ X \longrightarrow BA_{\text{Aut}}(F) \xrightarrow{A \text{ fibration of Ken complexes}} \]

This implies Lemma \( \bullet \).

**Details:** What is \( \text{Aut}(F) \)?

\[ \text{Map}(F, F) \xrightarrow{\text{Map}_\text{Aut}(F, F)_n} \xrightarrow{\Omega^0 \text{-shto}} (\Delta_n \times F, F) \]
Monoid (using $\Delta_n \to \Delta_n \times \Delta_n$)

$\text{Aut}(F) =$ sub-monoidal group of all invertible elements always forgetful (Moore)

**Principal bundles** for $G$ a monoidal group

Bundles with the structure of $G$ acting on $G$ as a torsor (by, say, left multiplication)

(we ask the top row of $\otimes$ to also preserve the $-$ action)

Principal bundles are classified by $BG$. 
Theorem: \( \{ \text{classes of } G \text{-principal bundles on } X \} \overset{f^*}{\rightarrow} \) 

\[ \begin{array}{c}
[X, BG] \\
\uparrow \\
B(G, h, \ast) \\
\downarrow \\
B^G = B(h, h, \ast)
\end{array} \]

Kan complex

\[ \begin{array}{c}
\text{simpler, all} \\
\uparrow \\
\text{up to strong \(~ \) } \\
\text{c-equivariant} \\
\uparrow \ast
\end{array} \]

Proof (sketch):

\[ \begin{array}{c}
\text{E} \longrightarrow B(G, h, \ast) \\
\downarrow \quad \\
X \longrightarrow B^G
\end{array} \]
For a bundle with fiber $F$ which is not principal. (Note: If $F$ has further structure, replace $\text{Aut}(F)$ by the group preserving that structure.)
If $p : E \to X$ is a bundle with fiber $F$, we need to replace it by a principal $\text{Aut}(F)$-bundle.

Recall the diagram $\mathcal{D}$:

$\Delta_n \times F \to \Delta_n \times X$

Over $\Delta_n \to X$, take $\text{Iso}_{\Delta_n}(\Delta_n \times F, \Delta_n \times X)$, the set of all these monomorphisms - for each over $\text{Aut}(F)_\Delta$.
total space of
the associated
principal bundle
to \( p : E \to X \).

\[
\begin{align*}
\text{DONE (with model structure on}
\text{simplicial set)}
\end{align*}
\]

Next, \( \text{simplicial sheaves on a Grothendieck site} \)

\( (\text{via covering sieves}) \)

\( \text{on sites which have pullbacks of} \)
Covers.

(Our case: smooth, separated, Noetherian schemes / Beck, Nisnevich topology.)

Pre sheaves = Contravariant functors

Sheaves $\rightarrow X_i \to X$

\[
\mathcal{B}(X_i, X, x_j) \leq \mathcal{B}(X_i) \leq \mathcal{B}(X)
\]

forget: sheaves $\rightarrow$ pre sheaves
\[ P(X) = \lim_{\text{co vers}} \left( \lim_{X_i \to X} P(X_i) \right) \]

\[ P(X) = \lim_{\text{co vers}} P(X_i) \]

\[ \text{may not be injective} \]

\[ \text{if injective: } \]

\[ P \text{ is called separated} \]

$L^P$ is separated if $P$ is separated, then $L^P$ is a sheaf.

$L^2 : \text{Presheaves} \to \text{Sheaves}$

is the left adjoint to the forgetful functor.