Hodge structure $A$ on an abelian group $H\alpha$

and a decomposition

$$H : = \mathbb{Z} \otimes \mathbb{C} = \bigoplus_{n} H^{p, q}_{\mathbb{C}}$$

$$H^{p, q} = H^{q, p}.$$

For a compact complex manifold $X$, the complex-valued de Rham complex is double-graded

$$0 \rightarrow \Omega^{n, 0} \rightarrow \Omega^{n, 1} \rightarrow \cdots$$
Recall the Hodge theorem for a compact Riemann manifold $X$:

The Laplace–Beltrami operator

A differential form $\alpha$ is called harmonic if $\Delta \alpha = 0$. 

Theorem: $H^k(X) \cong \text{Ker} \, \Delta^d$. □

Complex manifold: $\Delta^d, \Delta \bar{\partial}, \Delta \partial$

$\Delta^d = \Delta^2 + \Delta \bar{\partial}$

Kähler manifold: $\Delta \partial = \Delta \bar{\partial}$, from Hodge's lemma.

Harmonic forms $= \sum_1 \text{harmonic forms in degree } i$.

$H^k(X) = \bigoplus_{p + q = k} H^{p,q}$. 

$\omega \mapsto \omega^2$
\[ \therefore H^m X \text{ is a pure Hodge structure of weight } m, \]
\[ H^m = H^m(X; \mathbb{C}) \]

\text{ingular}

Example: Elliptic curve \( \mathbb{C}/\langle 1, \tau \rangle \), \( \text{Im } \tau > 0 \).

In dimension 1, the harmonic forms are integrals, integrals are called \( \text{periods} \)

\[ H^{1,0} = \langle dz \rangle \]
\[ H^{0,1} = \langle d\overline{z} \rangle \]

\( \tau \cdot \begin{pmatrix} 1 & \tau \\ \overline{\tau} & 1 \end{pmatrix} \)

\[ \tau \begin{pmatrix} \tau \\ \overline{\tau} \end{pmatrix} \]
compact Kähler manifold is smooth, projective variety over $\mathbb{C}$. What happens for any (quasi-)projective variety over $\mathbb{C}$?

Deligne: Mixed Hodge structure.

- An abelian group $H_2$
- An increasing filtration $\nu$, $H_2 = H_2 \oplus Q$(finite) (weight filtration)
- A decreasing filtration $F^\mu$ on $H = H \oplus C$ (Hodge filtration)

Condition: $gr_{\nu} H_2$ is a pure Hodge structure of
cyclic resolution of

unrelated

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differential with logarithmic singularities: \( \frac{d+}{d} \).

For singular quasi-projective varieties:

resolve nonpliorically, use simplicial techniques.

by smooth varieties.

One particular mixed Hodge structure: \( Q(m) \)

\[ Q(m) = \langle (2\pi i)^m \rangle \] has weight \( 2m \).

Compact Kähler case

\[ Q(m, c) = Q(m)^{m/m} \]
Deligne cohomology: in the smooth case, put the weight and Hodge filtration directly on forms.

Take twist

\[ H^i_D(X; \mathbb{Q}(j)) = \operatorname{Ext}^{i-j}_{\text{mixed}}(\mathbb{Q}(j), \Omega^n X) \]

Berthelot

Hyper cohomology - double complex

Very briefly, there is an analogous situation in
etale cohomology.

Sheaf cohomology in the etale site $X/K$

\[ \mathbb{H}^i_{et}(X, \mathcal{F}) = \text{Ext}^i_{\text{sheaves}}(\mathbb{Z}, \mathcal{F}) \]

\[ \text{char } k \neq l \]

"small" etale topos $\mathcal{X}$

\[ \mathbb{Z}/l(\mathcal{X}) : \mathbb{Z}/l(1) = \mu_l \subset l^{th} \text{ root of unity in } \overline{k} \]

Analogous to Deligne cohomology.

Analogous to Hodge structure?

\[ \mathbb{H}^i_{et}(X, \mathcal{F}) \] a representation
\[ X \times \text{geck} \Rightarrow \text{etak} \rightarrow \text{Cal} (k) \]

\[ \text{chart} \text{~} \]

Grothendieck : crystalline cohomology with vector

\[ X / \mathbb{F}_p \]

try to lift \( X \) to \( \mathbb{Z}_p \)

smoothly.

\[ \text{H} \text{or} (\text{left of } X) \]

---- p-adic Hodge theory.

--- Back to Voevodsky's theory

We have defined \( A^1 \)-homotopy category.
model structure of $\mathcal{A}^1$-spaces: weakly reduced
Wirthmühle have now
smooth pointed
Woothenen scheme / Beck.

cohomologies: section-wise injections
equivalences: $\mathcal{A}^1$-local equivalences
\{ $\mathcal{A}^1$- and $\mathcal{P}$-local

fibrations: determined.

Now we will define the stable $\mathcal{A}^1$-homotopy category
analogy of spectra in the $A'$-category.

Jardine: Motivic symmetric spectra
- local homotopy theory (localization)

\[ T = \prod_{\text{simplicial}} 
\begin{align*}
T &= \frac{\mathcal{A}}{\mathcal{A} \setminus \{0\}} \\
\end{align*}
\]

Jardine spectra: sequences of based $A'$-spaces $X_n$, bonding maps:
\( T \land X_n \rightarrow X_{n+1} \)

(Adjoint SL? of \( T \land i \) have no assumptions, mean)

From my point of view: perspective)