Math 215
Homework Set 1: §§13.1 – 13.4
Winter 2008

Most of the following problems are modified versions of homework problems from your text book *Multivariable Calculus* by James Stewart.

13.1a. Prove the Pythagorean theorem. (Hint: We do not expect you to produce an original proof. Rather, we expect you to find[1] a solution, understand the solution, and reproduce the solution. Please properly acknowledge all references you use. Also note that a proof which uses $\sin$ or $\cos$ probably is not a proof. Also, handing in a copy of something you find in cyberspace is not acceptable.)

13.1b. Sketch the surface in $\mathbb{R}^3$ represented by the equation

$$4x - 2y + z - 4 = 0.$$

13.1c. Find an equation for the set of all points equidistant from the points $S = (2, 1, 3)$ and $T = (-2, -2, -2)$. Describe the set.

13.1d. Describe, in words, the region in $\mathbb{R}^3$ represented by the inequality

$$x^2 + y^2 - 16 \geq 0.$$

13.2a. Molly Brown is walking due north on the deck of the Titanic at a speed of 3 km/hr. The Titanic is moving due west at a speed of 30 km/hr. Find Molly's velocity relative to the surface of the water.

13.2b. Suppose $\mathbf{a} = \langle -1, 1, 2\rangle$ and $\mathbf{b} = \langle 2, -1, -1\rangle$. Sketch each of the following quantities.

(a)
$$\mathbf{a} + 4\mathbf{j}$$

(b)
$$2\mathbf{a} - 3\mathbf{b}$$

13.3a. Suppose $\mathbf{u}$ and $\mathbf{v}$ are vectors. We define $\operatorname{orth}_{\mathbf{u}} \mathbf{v}$ to be the difference

$$\mathbf{v} - \operatorname{proj}_{\mathbf{u}} \mathbf{v}.$$

(a) Show that
$$\mathbf{v} = \operatorname{proj}_{\mathbf{u}} \mathbf{v} + \operatorname{orth}_{\mathbf{u}} \mathbf{v}.$$

(b) Show that $\operatorname{proj}_{\mathbf{u}} \mathbf{v}$ is orthogonal to $\operatorname{orth}_{\mathbf{u}} \mathbf{v}$.

(c) Compute and sketch the vectors $\operatorname{proj}_{\mathbf{u}} \mathbf{v}$ and $\operatorname{orth}_{\mathbf{u}} \mathbf{v}$ for $\mathbf{u} = \langle 2, 5, 7\rangle$ and $\mathbf{v} = \langle 1, 5, 8\rangle$.

13.3b. Compute the three angles, correct to the nearest tenth of a radian, of the triangle with vertices $(1, 2, 3)$, $(3, 5, -1)$, and $(-2, -3, -4)$. List your answers from least to greatest.

13.4b. Suppose $\mathbf{a} = \langle -1, 3, -2\rangle$ and $\mathbf{b} = \langle 2, -1, -1\rangle$. Compute

$$\mathbf{a} \times \mathbf{b}.$$

Sketch $\mathbf{a}$, $\mathbf{b}$, and $\mathbf{a} \times \mathbf{b}$.

13.4c. Find two unit vectors orthogonal to $\langle 1, 2, 4\rangle$. Can you find others?

[1] For example, you may wish to do a bit of research in a library.