Math 215
Homework Set 10: §§17.7 – 17.8
Winter 2008

Most of the following problems are modified versions of the recommended homework problems from your text book *Multivariable Calculus* by James Stewart.

17.7a A fluid with density 45 grams per cubic centimeter flows with velocity

$$\mathbf{v}(x, y, z) = \langle y, 1, z\rangle.$$

Find the rate of flow outward through the paraboloid $z = 16 - (x^2 + y^2)/9$ for $x^2 + y^2 \leq 144$.

17.7b Find the flux of $\mathbf{F}(x, y, z) = \langle y, 2x, 3z\rangle$ across the surface of the cube with vertices $(\pm 1, \pm 1, \pm 1)$.

17.7c Find the center of mass of the hemisphere given by the equations

$$x^2 + y^2 + z^2 \leq 25;\ y \leq 0.$$

Assume the density of the hemisphere is constant.

17.7d Find the flux of $\mathbf{F}(x, y, z) = \langle xy, y^2, z\rangle$ across the surface S where S is the part of the paraboloid $z = 2x^2 + 2y^2$ below the plane $z = 2$ with downward orientation.

17.8a. Verify that Stokes' Theorem is true for the vector field

$$\mathbf{F}(x, y, z) = \langle x, z^2, y^2\rangle$$

for the surface that lies on the paraboloid $y = x^2 + z^2$ and in the half-space $\{(x, y, z) \,|\, y \leq 1\}$. We assume that the surface is oriented in the positive direction of the y-axis.

17.8b. Suppose S is a surface with boundary C. Show that if $\mathbf{v}$ is a fixed vector and $\mathbf{F}(x, y, z) = \langle x, y, z\rangle$, then

$$2\iint_S \mathbf{v} \cdot d\mathbf{S} = \int_C (\mathbf{v} \times \mathbf{F}) \cdot d\mathbf{r}.$$

17.8c. Suppose $\mathbf{F}$ is a vector field. Consider two surfaces S and S' which possess the same boundary C. Use sketches (note plural) to describe how S and S' must be oriented so that

$$\iint_S \operatorname{curl}(\mathbf{F}) \cdot d\mathbf{S} = \iint_{S'} \operatorname{curl}(\mathbf{F}) \cdot d\mathbf{S}'.$$

17.8d. Suppose $\mathbf{F}$ is a vector field. Expanding upon the above problem: Suppose that S is a closed surface (that is, S is a boundary for a region of space). Show that

$$\iint_S \operatorname{curl}(\mathbf{F}) \cdot d\mathbf{S} = 0.$$