

**Math 215**  
**Homework Set 4: §§15.5 – 15.8**  
**Winter 2008**

Most of the following problems are modified versions of homework problems from your text book  
*Multivariable Calculus* by James Stewart.

- 15.5a. Because you will probably actually need to use these equations at some point in your academic career, please do Problems 45, 53, and 53 of §15.5 of Stewart's *Multivariable Calculus*.
- 15.5b. The temperature at a point  $(x, y)$  on the Michigan football field is  $T(x, y)$  measured in degrees Fahrenheit. Alan Mitchell runs so that his position after  $t$  seconds is given by  $x = 26 + \sqrt{25 + 3t^2}$  and  $y = 14 + t/5$  where  $x$  and  $y$  are measured in yards. The temperature function satisfies  $T_x(36, 15) = .3$  and  $T_y(36, 15) = .1$ . How fast is the temperature rising along Carter's path after 5 seconds?
- 15.5c. Suppose that the equation  $G(s, t, u) = 0$  implicitly defines each of the three variables  $s$ ,  $t$ , and  $u$  as functions of the other two:  $s = x(t, u)$ ,  $t = y(s, u)$ , and  $u = z(s, t)$ . If  $G$  is differentiable and  $G_s$ ,  $G_t$ , and  $G_u$  are all nonzero, show that

$$1 = -\frac{\partial u}{\partial s} \cdot \frac{\partial s}{\partial t} \cdot \frac{\partial t}{\partial u}.$$

- 15.6a. Use the table for wave heights in Problem 4 of §15.3 of Stewart's *Multivariable Calculus* to estimate the value of  $\mathbf{D}_{\mathbf{u}}f(40, 15)$ , where  $\mathbf{u} = (\mathbf{i} - \mathbf{j})/\sqrt{2}$ .
- 15.6b. Find the directional derivative of  $f(x, y, z) = x^2 + 3y^2 - 2z^2$  at the point  $(3, 5, 4)$  in the direction of the origin.
- 15.6c. Show that a differentiable function  $f$  decreases fastest at a point  $P$  in the direction opposite the gradient vector at  $P$ . In which direction is the function  $f(x, y, z) = x^2 + 3y^2 - 2z^2$  decreasing the fastest at the point  $(3, 5, 4)$ ?
- 15.6d. After copying the figure from problem 36 of §15.6 in Stewart's *Multivariable Calculus*, please do the problem.
- 15.6e. After copying the figure from problem 38 of §15.6 in Stewart's *Multivariable Calculus*, please do the problem.
- 15.6f. Find the points on the ellipsoid  $2x^2 + y^2 + 3z^2 = 6$  where the tangent plane is parallel to the plane  $4x + y + 6z = 7$ .
- 15.7b. Find the shortest distance from the point  $(2, 2, 1)$  to the plane given by the equation  $2x - 3y + z = 7$ .
- 15.7c. Consider the function  $f(x, y) = -3ye^x + y^3 + e^{3x}$ . Find and classify the critical point of  $f$ . Does the function  $f$  obtain an absolute extremum at the critical point? You may wish to use MAPLE to help you answer the latter question.
- 15.7d. You are to design a rectangular building to house the University's art collection. Per square yard, the cost for the foundation is four times the cost of the material for the walls which is twice the cost of the material used to construct the roof. If the university has  $D$  dollars to spend and the cost of the material for the roof is  $d$  dollars per square foot, give the dimensions (in yards) which maximize the volume of the building.
- 15.7e. Find the extreme values for the function  $f(x, y) = 2x + y^2 - e^x$  on the unit disc  $D = \{(x, y) \mid x^2 + y^2 \leq 1\}$ . You may wish to use MAPLE to solve some of the relevant equations.