Math 215
Homework Set 7: §§16.7 – 17.2
Winter 2008

Most of the following problems are modified versions of the recommended homework problems from your text book *Multivariable Calculus* by James Stewart.

16.8a. Find the volume of one of the smaller wedges cut from a sphere of radius 27 by two planes that intersect along a diameter at an angle of $\pi/6$.

16.8b. Find the volume and center of mass of the solid that lies above the cone $z = 3\sqrt{x^2 + y^2}$ and below the sphere $x^2 + y^2 + z^2 = 9$. Assume that the density of the solid is constant.

16.8c. Find the volume of the solid that lies above the cone $\varphi = \pi/3$ and below the sphere $\rho = 9\cos(\varphi)$.

16.8d. Evaluate

$$\iiint_B (3x^2 + 3y^2 + 3z^2)\, dV$$

where B is the ball of radius 13 centered at the origin.

16.8e. Find the center of mass of a solid hemisphere of radius 5 if the density at any point in the hemisphere is proportional to the point's distance from the *base* of the hemisphere.

16.8f. Do Problems 27–28 of §16.7 39 and 40 of §16.8 in Stewart's *Multivariable Calculus*.

17.1a. Do Problems 11–14 of §17.1 in Stewart's *Multivariable Calculus*.

17.1b. Do Problems 15–18 of §17.1 in Stewart's *Multivariable Calculus*.

17.1c. Do Problems 29–32 of §17.1 in Stewart's *Multivariable Calculus*.

17.2a. Do Problem 47 of §17.2 in Stewart's *Multivariable Calculus*.

17.2b. Farmer Pickles wants Bob to paint the circular fence which encloses his sunflower field. If the parametric equations $x = 13\cos(\theta)$ and $y = 13\sin(\theta)$ describe the base of the fence (in yards) and the height of the fence is given by the equation $h(x, y) = 11 + (2x - y)/6$, then how many gallons of paint with Bob need to complete the project. Assume that one gallon of paint covers three hundred square feet of fence.

17.2c. Find the mass and the center of mass of a wire loop in the shape of a helix (measured in cm: $x = t$, $y = 4\cos(t)$, $z = 4\sin(t)$ for $0 \le t \le 2\pi$), if the density (in grams/cm) of the wire at any point is equal to the square of the distance from the origin to the point.

17.3a. Do Problem 33 of §17.3 in Stewart's *Multivariable Calculus*.

EXTRA CREDIT. Suppose we have an ellipse parameterized by $x = a\cos(\theta)$ and $y = b\sin(\theta)$ with a and b positive. This is the ellipse described by the equation $x^2/a^2 + y^2/b^2 = 1$. It is known that the area of the ellipse is πab. The following argument shows that there is only one positive real number. What is wrong with the argument?

Suppose a and b are positive real numbers. We shall compute the area of the ellipse parameterized by $x = a\cos(\theta)$ and $y = b\sin(\theta)$ for $0 \leq \theta \leq 2\pi$ by using polar coordinates. Since $r^2 = x^2 + y^2$, we have $r^2 = a^2\cos^2(\theta) + b^2\sin^2(\theta) = b^2 + (a^2 - b^2)\cos^2(\theta)$. Thus,

$$\begin{aligned}
\pi ab &= \int_0^{2\pi} \int_0^{\sqrt{b^2+(a^2-b^2)\cos^2(\theta)}} r\,dr\,d\theta \\
&= \frac{1}{2}\int_0^{2\pi} (b^2 + (a^2 - b^2)\cos^2(\theta))\,d\theta \\
&= \frac{1}{2}\Big|_0^{2\pi} (b^2\theta + (a^2 - b^2)(\theta/2 + \sin(2\theta)/4) \\
&= \frac{(a^2 + b^2)\pi}{2}
\end{aligned}$$

So, $2ab = a^2 + b^2$, and hence $(a - b)^2 = 0$ which means $a = b$. Since a and b were arbitrary positive real numbers, we conclude that there is only one positive real number.