M419 - Linear Spaces and Matrix Theory, Fall 2013

Assignment # 4.
Due: Thursday, October 3, 2013.

From the text (Strang, Linear Algebra and Its Applications, 4th Edition)

Section 2.2 – Problems 4, 10, 24, 36, 42, 68.
Section 2.3 – Problems 2, 14, 16, 24, 34.

Additional Problem: Interpolation

Interpolation is a method for producing a smooth curve which passes through a given set of data points.

A. Consider the set of points in the $x - y$ plane (0, 0), (1, 4), (2, 3), and (3, 0). Find a cubic polynomial $p(x)$ that passes through these points, satisfying $p(0) = 0$, $p(1) = 4$, $p(2) = 3$, and $p(3) = 0$. Sketch this curve; or, if you have a computer graphics program at hand, print out a graph of the curve.

B. Prove that, for any real numbers $a$, $b$, $c$, and $d$, there exists a cubic polynomial $p$ whose graph passes through the points $(0, a)$, $(1, b)$, $(2, c)$, and $(3, d)$, and that this polynomial is unique.

C. Suppose we only had three points, $(0, a)$, $(1, b)$, and $(3, c)$ to work with. Carry out an analysis similar to the above to answer the questions: Is there always a cubic polynomial whose graph passes through the three points? Is this cubic polynomial unique?