
Math 454 - Final Formula Sheet

1. Rayleigh Quotient:

$$\lambda = \frac{-p\phi \frac{d\phi}{dx} \Big|_a^b + \int_a^b \left[p \left(\frac{d\phi}{dx} \right)^2 - q\phi^2 \right] dx}{\int_a^b \phi^2 \sigma dx}$$

2. Some Useful Trigonometric Identities:

$$2 \sin(\alpha) \sin(\beta) = \cos(\alpha - \beta) - \cos(\alpha + \beta)$$

$$2 \sin(\alpha) \cos(\beta) = \sin(\alpha - \beta) + \sin(\alpha + \beta)$$

$$2 \cos(\alpha) \cos(\beta) = \cos(\alpha - \beta) + \cos(\alpha + \beta)$$

3. Fourier sine series:

$$f(x) \sim \sum_{n=1}^{\infty} B_n \sin\left(\frac{n\pi x}{L}\right), \quad B_n = \frac{2}{L} \int_0^L f(x) \sin\left(\frac{n\pi x}{L}\right) dx, \quad n = 1, 2, 3, \dots$$

4. Fourier cosine series:

$$f(x) \sim \frac{A_0}{2} + \sum_{n=1}^{\infty} A_n \cos\left(\frac{n\pi x}{L}\right), \quad A_n = \frac{2}{L} \int_0^L f(x) \cos\left(\frac{n\pi x}{L}\right) dx, \quad n = 0, 1, 2, 3, \dots$$

5. Fourier series:

$$f(x) \sim \frac{A_0}{2} + \sum_{n=1}^{\infty} \left[A_n \cos\left(\frac{n\pi x}{L}\right) + B_n \sin\left(\frac{n\pi x}{L}\right) \right]$$

$$A_n = \frac{1}{L} \int_{-L}^L f(x) \cos\left(\frac{n\pi x}{L}\right) dx, \quad n = 0, 1, 2, 3, \dots$$

$$B_n = \frac{1}{L} \int_{-L}^L f(x) \sin\left(\frac{n\pi x}{L}\right) dx, \quad n = 1, 2, 3, \dots$$

6. Parseval's Identity:

$$\int_a^b f^2(x)\sigma(x)dx = \sum_{n=1}^{\infty} a_n^2 \int_a^b \phi_n^2(x)\sigma(x)dx$$

where

$$f(x) = \sum_{n=1}^{\infty} a_n \phi_n(x)$$

7. Bessel's Functions:

$$r^2 f''(r) + r f'(r) + (\lambda r^2 - m^2) f(r) = 0, \quad m = 0, 1, 2, \dots$$

$$f(r) = c_1 J_m(\sqrt{\lambda}r) + c_2 Y_m(\sqrt{\lambda}r)$$

First Kind: $J_0(0) = 1$, $J_m(0) = 0$ for $m > 0$

Second Kind: $|Y_m(z)| \rightarrow \infty$ as $z \rightarrow 0$ for $m \geq 0$