

## M572 - Numerical Methods for Scientific Computing II - 2008

Assignment # 1.

Due: Thursday, January 17, 2008.

### 1. Observed Order of Accuracy

I. Consider  $u(x) = e^x$ ,  $x_0 = 0$ . Compute  $D_+u(x_0)$ ,  $D_0u(x_0)$ ,  $D_3u(x_0)$ , using  $h = 0.1, 0.05, 0.025, 0.0125$ . Compute the error. For each finite difference approximation, construct a table of:  $h$ , error, error/ $h$ , error/ $h^2$  and error/ $h^3$ , confirm the expected order of accuracy. Compute the constants  $C$  and compare with theoretical values. Plot  $\log(\text{error})$  against  $\log h$ .

II. Perform Richardson's extrapolation on the values of  $D_+u(x_0)$ , and  $D_0u(x_0)$ .

### 2. Roundoff Errors

Consider again  $u(x) = e^x$ ,  $x_0 = 0$ . Compute  $D_+u(x_0)$  for a sequence of increasingly small  $h$  values, for example,  $h = 10^{-n}$ . Plot the error against  $h$  and explain your observations.

### 3. Finite Differences using interpolating polynomials

Let  $p(x)$  be the interpolating polynomial to  $u(x)$  at the points  $x_0$ ,  $x_0 - h$  and  $x_0 - 2h$ . Show that  $p'(x_0)$  gives exactly the 3-point one-sided difference approximation  $D_2(x_0)$  derived in class.

### 4. High order approximations using Richardson's extrapolation

Consider the backward difference approximation  $D_-u(x_0)$  based on a step size  $h$ , and on a step size  $2h$ . Show that Richardson's extrapolation on the two approximations gives  $D_2(x_0)$ .

### 5. Finite difference approximations with unequally spaced points.

I. Use the method of undetermined coefficients to derive the 3-point difference approximation to  $u''(x_0)$  using the unequally spaced points  $u(x_0 - h_1)$ ,  $u(x_0)$  and  $u(x_0 + h_2)$ .

$$u''(x_0) \approx \frac{2u(x_0 - h_1)}{h_1(h_1 + h_2)} - \frac{2u(x_0)}{h_1h_2} + \frac{2u(x_0 + h_2)}{h_2(h_1 + h_2)}$$

Comment on the accuracy of the approximation.

II. Use this formula to approximate the derivative of  $e^x$  at  $x_0 = 0$  when

- $h_1 = 0.05$  and  $h_2 = 0.1$
- $h_1 = 0.005$  and  $h_2 = 0.01$
- $h_1 = 0.0005$  and  $h_2 = 0.001$

Compare with the exact answer. By what factor is the error decreasing at each step?

III. Confirm that the approximation reduces to the centered formula when  $h_1 = h_2 = h$  and use it to compute the second derivative with (a)  $h = 0.1$  (b)  $h = 0.01$  and (c)  $h = 0.001$ . By what factor is the error decreasing at each step?

IV. Discuss the accuracy of centered vs. non-centered approximations.