

Introduction to Shimura varieties

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This course is a brief introduction to Shimura varieties and serves to provide some background knowledge for other lectures. Emphasis will be on PEL Shimura varieties, which have played increasing roles in arithmetic, rather than general Shimura varieties. A PEL Shimura variety is defined to be the parameter space for abelian varieties with endomorphism and level structures. The definition of integral models and basic properties will be explained. Along the way I will present examples such as 0-dimensional Shimura varieties, modular curves and $U(n-1, 1)$ -Shimura varieties. If time permits, the cohomology of Shimura varieties may be discussed. It would be helpful to be familiar with modular curves, especially their interpretation as moduli spaces for elliptic curves with additional structure.

Suggested reading:

1. Kottwitz, Robert E., *Points on some Shimura varieties over finite fields*. J. Amer. Math. Soc. 5 (1992), no. 2, 373-444. (especially section 5)
2. Harris, Michael; Taylor, Richard., *The geometry and cohomology of some simple Shimura varieties*. With an appendix by Vladimir G. Berkovich. Annals of Mathematics Studies, 151. Princeton University Press, Princeton, NJ, 2001. viii+276 pp. ISBN: 0-691-09090-4 (especially chapter 3)
3. Deligne, Pierre, *Travaux de Shimura*. (French) Séminaire Bourbaki, 23ème année (1970/71), Exp. No. 389, pp. 123-165. Lecture Notes in Math., Vol. 244, Springer, Berlin, 1971.
4. Milne, James, *Introduction to Shimura varieties*
<http://www.jmilne.org/math/xnotes/svi.pdf>