Consider the following function, where \( a \) is a positive constant and \( k \) is a constant.

\[
f(x) = \begin{cases} 
  a e^x + ea^x + x\pi + k & \text{for } x \geq 0 \\
  \frac{1}{\ln 5} 5^x + x + 3^e & \text{for } x < 0
\end{cases}
\]

1. Find \( k \) in exact form so that \( f \) is continuous at all points in its domain. Your answer may involve the constant \( a \).

2. Now, let \( a = 2 \) and assume the constant \( k \) is chosen so that \( f \) is continuous everywhere. Find a formula for the slope of the tangent line to the graph of \( f \) at each point \( x \). Are there any points where a well-defined tangent line fails to exist? Are there any points where \( f \) fails to be differentiable?

3. Again suppose \( a = 2 \). If \( f \) is modeling the growth (in thousands of individuals) of a colony of bacteria in the dorm fridge \( x \) days after break starts on midnight of March 1 (say, March 1 at 12:01 am) find the instantaneous rate of growth of the colony at midnight on March 8 (March 8 at 12:01 am). Be sure to include units.

4. Use your answer to 3 to estimate the amount by which the population of bacteria increases from March 8 at midnight to March 8 at noon, when you arrive back at the dorm. Be sure to include units.