Quiz 1A

A. State the *precise* definition of a **linear transformation** $\mathbb{R}^n \xrightarrow{T} \mathbb{R}^m$.

B. Which of the following are linear transformations?

1. The mapping $\mathbb{R}^2 \to \mathbb{R}^2$ which dilates by a factor of 2.
2. The mapping $\mathbb{R}^3 \to \mathbb{R}^3$ which rotates counterclockwise around the z-axis by 90^0.

C. Let A be an $m \times n$ matrix and let $\vec{e}_j$ be the j-th standard unit vector in $\mathbb{R}^n$. Describe exactly the matrix product $A\vec{e}_j$ in terms of the rows and/or columns and/or entries of A.

A Crucially Important Theorem: *If* $\mathbb{R}^n \xrightarrow{T} \mathbb{R}^m$ *is a linear transformation, then there exists a* **unique** *matrix* A *such that for all* $\vec{x} \in \mathbb{R}^n$*, we have*

$$T(\vec{x}) = A\vec{x}.$$

What are the dimensions of A? Can you describe how to find A? Can you prove this crucial theorem?