

217 Quiz 7

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1. Find the QR factorization of $\begin{bmatrix} 0 & 1 & 1 \\ 0 & 1 & 9 \\ 0 & 1 & 9 \\ 0 & 1 & 1 \\ 1 & 2 & 1 \end{bmatrix}$.

Solution note: $\begin{bmatrix} 0 & 1/2 & -1/2 \\ 0 & 1/2 & 1/2 \\ 0 & 1/2 & 1/2 \\ 0 & 1/2 & -1/2 \\ 1 & 0 & 0 \end{bmatrix} \cdot \begin{bmatrix} 1 & 2 & 1 \\ 0 & 2 & 10 \\ 0 & 0 & 8 \end{bmatrix}$

2. Let V be the subspace of \mathbb{R}^5 spanned by the columns of the matrix in (1). Define V^\perp in precise set builder notation, and compute its dimension.

Solution note: $V^\perp = \{\vec{x} \in \mathbb{R}^5 \mid \vec{x} \cdot \vec{v} = 0 \ \forall \vec{v} \in V\}$.

3. With

V as in (2), find the projection of the vector $[1 \ 0 \ -1 \ 0 \ 2]^T$ onto V .

Solution note: Let $\vec{u}_1, \vec{u}_2, \vec{u}_3$ be the orthonormal vectors in (1). The projection is $(\vec{v} \cdot \vec{u}_1)\vec{u}_1 + (\vec{v} \cdot \vec{u}_2)\vec{u}_2 + (\vec{v} \cdot \vec{u}_3)\vec{u}_3$ which is

$$2\vec{u}_1 + 0\vec{u}_2 - \vec{u}_3 = \begin{bmatrix} 1/2 \\ -1/2 \\ -1/2 \\ 1/2 \\ 2 \end{bmatrix}.$$

4. What vector in V is the closest to $[1 \ 0 \ -1 \ 0 \ 2]^T$?

Solution note: Same answer as (3).