Math 217: Quiz 8
Professor Karen Smith

1. **Define** what it means that a linear transformation $\mathbb{R}^n \xrightarrow{T} \mathbb{R}^n$ is **orthogonal** (your definition should match the definition in the book and on the worksheets).

2. Prove that if $\mathbb{R}^n \xrightarrow{T} \mathbb{R}^n$ is an orthogonal linear transformation, then $T\vec{v} \cdot T\vec{w} = \vec{v} \cdot \vec{w}$ for all $\vec{v}, \vec{w} \in \mathbb{R}^n$. [Hint: Consider $\vec{v} + \vec{w}$.]

(OVER!)

3. Suppose you are given a vector $\vec{b} \in \mathbb{R}^4$ and a 4×3 matrix A with QR factorization

$$A = QR = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & -1 \\ 1/\sqrt{2} & 0 & 0 \\ 1/\sqrt{2} & 0 & 0 \end{bmatrix} \begin{bmatrix} 2 & 3 & -1 \\ 0 & 1 & 0 \\ 0 & 0 & 2 \end{bmatrix}.$$

1. Let $\vec{u}_1, \vec{u}_2$ and $\vec{u}_3$ be the columns of Q. Suppose that $[\vec{u}_1 \ \ \vec{u}_2 \ \ \vec{u}_3 \ \ \vec{b}]$ has rank 4. Does $A\vec{x} = \vec{b}$ have a solution? Explain.
2. Write the third column of A as a linear combination of $\vec{u}_1, \vec{u}_2, \vec{u}_3$.
3. Suppose $\vec{b} \cdot \vec{u}_1 = \sqrt{2}$, and $\vec{b} \cdot \vec{u}_2 = \vec{b} \cdot \vec{u}_3 = 2$. Find a vector $\vec{b}^*$ such that the solutions of the system $A\vec{x} = \vec{b}^*$ are the least squares solutions of $A\vec{x} = \vec{b}$.
4. Find the least squares solutions of $A\vec{x} = \vec{b}$. [Hint: If you are clever, there should be no row-reducing.]