Math 512. Quiz 1 Friday September 9, 2011.

1. Let \( f \) and \( g \) be two linear transformations of a vector space. Prove that \( f \circ g \) is also a linear transformation.

2. Let \( \Box \) be a binary operation on a set \( S \). Prove that if \( \Box \) has an identity element, then it is unique. [Hint: if there are two identities, consider their product.]

3. Let \( A = \begin{pmatrix} a_1 & \ldots & a_n \end{pmatrix} \) and let \( B = \begin{pmatrix} b_1 \\ \vdots \\ b_n \end{pmatrix} \), where \( n \) is some integer \( \geq 2 \).

   a.) Compute the matrix products \( AB \) and \( BA \).

   b). There is a very simple formula for the determinant of \( BA \), and a simple justification for it using one of the main theoretical properties of the determinant. What is this formula and its proof?