This Daily Update will be written as soon as possible after I finish teaching at 11:30. On many days, that could mean after kids go to bed. It is basically a log of what we did, for example, by listing the relevant page numbers in the notes, but will also will correct errors and make clarifications, and provide announcements.

Tuesday September 24: We had a rare lecture, just to get everyone on the same page with localization, or attempt to. We then worked at the board trying to compute the fiber over a point for a map of Spectra. Your minimum homework is to complete the Worksheet Problems 1, 2, 3. Problem 1 is a review of Thursday’s worksheet, or equivalently, of today’s lecture—it’s just a list of questions to help you test your understanding. Problem 3 is the proof of the Theorem on fibers, and Problem 2 is a bunch of easy practice problems computing fibers. Of course, A students will also want to practice further, problems 4 and 5 are more interesting fiber computations. Problem 6 can be skipped for now, if you are struggling to keep up (I won’t put it on the quiz).

Thursday September 19: We worked on localization of rings, the first few problems were supposed to be review (or from the Reading; see Hochster’s notes starting around page 22) but students were pretty rusty on the topic, so it took longer than I expected. Still, most students got through Problems 1-4. The core material (for now) here is Problems 1-7. One thing that slowed some folks down on the earlier problems was insistence on a particular definition of “prime ideal.” In many cases, these would have been more straightforward with the more concrete/basic definition using elements (i.e., $P$ is prime if $xy \in P$ implies $x \in P$ or $y \in P$). Of course, a super useful and important fact (which some folks were trying to use) is that $P$ is prime if and only if $R/P$ is a domain. This point of view becomes easier to work with if we also take advantage of the universal properties of localization and quotients. To get better at this, be sure to do Problem 7 on today’s worksheet.

Problem 5 contains an important fact (that the radical of an ideal is the intersection of all primes containing it) as a corollary of the core idea in (4) describing the prime ideals in a localization $RU^{-1}$ as the primes in $R$ disjoint from $U$. Especially if you used this without proof on Quiz 1, you should make sure you can prove it!

Problem 6 also contains an important idea: Spec $(R \times S)$ can be identified with the disjoint union Spec $R \cup$ Spec $S$. Product rings seemed less familiar to a lot of students than I expected, so please practice by doing this exercise!

Assignment for Tuesday Sept 24: Focus on getting Problems 1-7 understood and written down. This is a good time to take stock of how your worksheet solutions are coming along. We will return to “Fibers” later...possibly as a lecture...we’ll see.

Tuesday September 17: We took Quiz 2. We then worked through the proof of Hilbert’s Nullstellensatz using a Worksheet in the case that $K$ is uncountable (for example, over $\mathbb{C}$). The proof was in Problems 3-6 (though 3 was done on the last worksheet), with the ingenious trick that makes it work Problem 4b. As is often the case with the hard theorems, the hard part comes down to clever linear algebra (or clever combinatorics or, according to my analyst friends, clever applications of the triangle inequality).

Assignment for Thursday Sept 19: Worksheet problems 1, 3 and 7 are conceptual core material on the algebra-geometry correspondence of Commutative Algebra, whereas 4, 5, 6 are the details of the proof of the Nullstellensatz. Problems 2, 8, 9, 10 are excellent practice for working with the concepts and could be seen on a future quiz or used in future worksheet problems. Problem 11 is easy but more abstract and can be revisited later if you are overwhelmed. Next time will we discuss localization, which should be mostly review from Math...
593. Read through page 28 of Hochster’s Fall 2017 Math 614 notes (link provided on our Math 614 homepage) if you haven’t already, and review the “universal properties” of localization and quotient rings from Math 593. I will collect Problem Set 1.

**Important note about grades:** The quizzes are supposed to be easy, and indeed, the Median score among folks with the prereqs was 7/8, in part because I was very picky. However, students without the prereqs did not fare as well. Please be aware that the course will be at the graduate level, and unlike "honors" courses at Michigan, grades will not be restricted to the A- to A+ range. I’m sorry that we won’t be able to slow down to accommodate students who are missing Alpha course material. However, Math 593 is a great class, with a great instructor!

**Thursday September 12:** We took Quiz 1. We began a worksheet on Algebraic Sets. Several groups proceeded nicely through Problem 4 (or more), which is the pace I believe is right for this graduate course if it is done carefully. But others struggled, for two different reasons. Some folks were careful, but uncertain about basic things like how to check an ideal is maximal or what quotient rings are. These students should seriously consider Math 593, especially if they want to pass qualifying exams or go to grad school. A few others blazed through pulling in fancy concepts but missing the basic point, writing things that don’t make sense or using nuclear bombs (eg, Hilbert’s Nullstellensatz) to prove special cases of simple Math 412/593 facts (like a polynomial ring in one variable has a division algorithm). To these students I say: Slow down and understand more deeply!

**Assignment for Tuesday Sept 17’s class:** Write up the (unstarred) worksheet problems through Problem 4, carefully, or through 6 or more, carefully, if you feel you fully understand and enjoy. I will return to the concepts in problems (5, 6, 7, 8) on a later worksheet in a slightly different way, so at a minimum make sure you get through 4. I know Problem Set 1 is also due next week.

**There is a high probability of a short (five minute) quiz next time on the most basic ideas from today’s worksheet.**

**Tuesday September 10:** Students did a great job going through the Worksheet on the Zariski Topology and the functor Spec. Most groups got through most of it. In writing up in your own words, the key problems to write up, at the very least, are 2, 4, 6ab, 7,11. These are the theoretical “theorems” of the class—the other problems are great for building intuition so they are also important if you really want to master commutative algebra. I strongly encourage you to sit down after class while things are fresh and write up as much as you can or as much as you are interested in. Latex is great, but handwritten is fine. Hint: you can take pictures of your work on the board.

**Assignment for Thursday Sept 12’s class:** Read through page 28 of Hochster’s Fall 2017 Math 614 notes (link provided on our Math 614 homepage). Try to focus on concretely understanding free modules, Hom, and what is localization, which should be review from Math 593. However, he has a lot of categorical framework around it which might make it less familiar. Start looking at Problem Set 1, which is posted on the website. I will collect your solutions next Thursday.

**Tuesday and Thursday September 3 and 5:** Eric Canton substituted. Tuesday he gave a brief overview on what is commutative algebra. Thursday, students worked on a worksheet on Noetherian ring. You should complete this for homework, and keep your work organized in a “Lab Notebook” that includes each worksheet with the date and your collaborators.

**Assignment for Tuesday Sept 10’s class:** Read through page 18 of Hochster’s Fall 2017 Math 614 notes (link provided on our Math 614 homepage). This includes an overview of what commutative algebra is about, and a
review of the language of category theory, which will be a useful framework for us as well. [Note: Hochster’s class met for 50 minute increments, so about three Hochster lectures per week is our pace, although we won’t follow his course exactly.]