1. Find the Galois group (over \( \mathbb{Q} \)) of the polynomial \( x^4 - 18x^2 + 77 \). Express each element explicitly in cycle notation as elements of the permutation group of the roots.

Factor \( x^4 - 18x^2 + 77 = (x^2 - 7)(x^2 - 11) \) so the roots are \( \pm \sqrt{7}, \pm \sqrt{11} \) and the splitting field is \( L = \mathbb{Q}(\sqrt{7}, \sqrt{11}) \). Since \( L = K(\sqrt{7}) \) where \( K = \mathbb{Q}(\sqrt{11}) \), there is an automorphism of \( L \) fixing \( K \) which sends \( \sqrt{7} \) to any other root of \( x^2 - 7 \) in \( L \). So there is an element \( \phi \in G \) swapping \( \pm \sqrt{7} \) and fixing \( \sqrt{11} \). Similarly, there is one swapping \( \pm \sqrt{11} \) and fixing \( \sqrt{7} \). Ordering the roots \( \sqrt{7}, -\sqrt{7}, \sqrt{11}, -\sqrt{11} \), these two elements of the Galois group are (12) and (34) in \( S_4 \). Then

2. Let \( G \) be the group \( S_n \) acting on the set \( \mathcal{X} = \{1, 2, 3, \ldots, n\} \) in the obvious way.

   (1) Define the \textbf{stabilizer} of an element \( x \in \mathcal{X} \).
   (2) Prove that there is only one orbit for this action of \( S_n \) on \( \mathcal{X} \).
   (3) Prove that the stabilizer of each \( x \in \mathcal{X} \) is isomorphic to \( S_{n-1} \).
   (4) If \( x \neq y \in \mathcal{X} \), can the stabilizers of \( x \) and \( y \) be the same?