Math 115 Classwork 13: Team Quiz

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Remember the donkey cart ride? Your team had found the following model for your instantaneous velocity over the one hour trip:

\[ v(t) = 0.01t^2 \text{ meters/min} \]

where \( t \) is measured in minutes. This was based on the data you collected:

\[
\begin{array}{c||c|c|c|c|c|c|c}
 t(\text{min}) & 0 & 10 & 20 & 30 & 40 & 50 & 60 \\
v(t)(\text{meters/min}) & 0 & 1 & 4 & 9 & 16 & 25 & 36 \\
\end{array}
\]

1. Graph your model, and label the relevant data points (or pull out the graph you made last week).

2. Express the total distance travelled on the one hour trip as an integral, being careful to use precise notation and units.

3. Write out, without computing, the left and right Riemann sums (with \( n = 6 \)) which estimate the integral in (2), including all units. What is \( \Delta t \)? Show the rectangles giving the right Riemann sums on your graph (and the left sum too if not too messy).

4. Now compute the left and right Riemann sums, explaining what they mean in the context of the notation and terminology of section 5.2. State in clear sentences exactly what these numbers mean: which integral are they estimating, and what is the interpretation in the context of the donkey trip? What assumptions underlie the estimates? Is either sum is an over or under estimate? Why? Why does increasing \( n \) lead to a more accurate estimate? What happens to \( \Delta t \) as \( n \) is increased?

5. Average your over and underestimates from (4) to get an estimated length (in meters) of the donkey cart trip. (Or recall your estimate from last week).

6. The senior math majors in the next donkey cart over have been recording distance travelled during the trip (instead of velocity) and have produced the following model for the distance travelled on the trip:

\[ s(t) = \frac{t^3}{300} \text{ meters} \]

at time \( t \) in minutes. Is their data consistent with yours? Is their model consistent with your model of \( v(t) \)? What is the precise mathematical relationship between the functions \( s(t) \) and \( v(t) \), expressed in math symbols?

7. Use the math majors’ model to compute the exact length in meters of the donkey cart trip. How does this compare to your estimate in 5?
8. What is the exact value of the integral \( \int_0^{60} v(t)dt \)?

9. Express the distance travelled in the second half of the trip as an integral, paying careful attention to the correct notation. Sketch a graph with a shaded area representing the exact distance traveled.

10. Use the seniors’ model to compute the exact distance covered in the second half of the trip.

11. Find the exact value of the integral in 9. Do left and right Riemann sum estimations to see whether your answer is reasonable.

12. Express the distance travelled during the middle third of the trip as an integral. Now evaluate the exact value of the integral using the seniors’ model from 6.

13. Express the distance travelled between \( t = a \) minutes and \( t = b \) minutes of the trip as an integral. Now evaluate the exact value of the integral (your answer will involve \( a \) and \( b \)).

Here come the goats!

14. The next day you set out with your team in a wagon pulled by goats. The velocity of the goat wagon is given by \( w(t) = \frac{2t}{3} + 5 \) meters per minute, after \( t \) minutes. Write an integral representing the length of the hour long goat-wagon trip in meters.

15. Write an integral representing the number of meters traversed during the first \( b \) minutes of the trip. Compute an exact value of this integral.

16. Can you find a formula for the function \( g(t) \) representing the length of the trip (in meters) after \( t \) minutes? What is the relationship between \( g \) and \( w \)? Relate this to your answer in 15 as well.

And the camels!

17. The camel cart travels at a velocity of \( f(t) \) meters per minute. Write an integral representing the total distance travelled (in meters) between time \( t = a \) and \( t = b \) minutes.

18. You are told also that the distance travelled by the camel cart is modeled by the function \( F(t) \) in meters, where \( t \) is the time travelled in minutes. Write a formula for the distance travelled between times \( a \) and \( b \) using \( F \).

19. What is the precise mathematical relationship (in math symbols) between \( F \) and \( f \)?

20. Is the following statement true or false? Explain:

\[
\int_a^b f(t)dt = F(b) - F(a).
\]