

**Math 217 Worksheet on Practicing Induction**  
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FOR EACH STATEMENT, VERIFY DIRECTLY THE FIRST FEW CASES. THEN PROVE THE GENERAL STATEMENT BY INDUCTION. BE SURE TO CLEARLY INDICATE THE STATEMENT  $P_n$  TO BE PROVED (USING MATHEMATICAL NOTATION AS MUCH AS POSSIBLE), THE BASE CASE, AND THE INDUCTIVE STEP.

1. For the matrix  $A = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$ ,  $A^n = \begin{bmatrix} 1 & n \\ 0 & 1 \end{bmatrix}$  for all natural numbers  $n$ .

2.  $1 + 2 + 3 + \cdots + n = \frac{1}{2}n(n + 1)$  for all natural numbers  $n$ .

4. Let  $A$  and  $B$  be  $2 \times 2$  matrices, and assume that  $A$  is invertible. Prove that  $(A^{-1}BA)^n = A^{-1}B^nA$  for all natural numbers  $n$ .

4. An  $n$  element set has exactly  $2^n$  subsets, for all natural numbers  $n$ .