Math 217 Daily Update Winter 2016 Section 5

This **Daily Update** will be written as soon as possible after I finish teaching at 1 pm. It will summarize what we did, correct errors and make clarifications, provide important announcements, such as information about upcoming quizzes, and occasionally offer some general advice. Make a habit of checking it frequently.

Monday April 18: We did some proof technique practice: reviewing standard techniques like showing vectors are linearly independent by assuming there is a relation, then showing it is trivial. We then worked on a TF sheet. I sent (and improved version) to you by email, with answers. It is also posted. EXAM THURSDAY AT 10:30. I will hold office hours Wednesday from 12 to 4 in the usual room (maybe we'll move outside....let's see). By then, I hope you will have 1) done all the practice exams, 2) finished the TF worksheet 3) gone over your homeworks to make sure you understand any mistakes; 4) reread the book chapters 4, 5 (especially 5.5), 7 and 8.1. 5). reread the documents on my webpage to make sure you know all definitions and theorems–the coordinates document might be especially useful; 6) redone all the worksheets and checked your answers against mine, especially those on B-matrices, inner product spaces, and eigen-everything. STUDY HARD! WE WERE PROMISED A HARD FINAL!

Friday April 15: Guest instructor and famed Math 217 teacher Hanna Bennet subbed! Thanks for your patience as I am travellng with my daughter so she can see some colleges before she chooses one by May 1. I will try to post that worksheet with answers.

Wednesday April 13: We first reviewed some proof techniques. Get the notes from a friend if you missed! Techniques: 1). showing T is injective by showing the kernel is zero (and this by taking some element in the kernel ...) 2). Showing one set X is contained in another Y by taking an arbitrary element in X and messing around to get it in Y; 3). Showing some x is $V^{\perp}$ by showing that $\langle x, v\rangle = 0$ for every $v \in V$; 4). Showing some element x in an inner product space is zero by showing $\langle x, x\rangle = 0$ (positive definitive property).

We then discussed the SPECTRAL THEOREM using a worksheet. Please finish it if you haven't.

For Friday: Lecturer Hannah Bennett subbing. It will be good! Finish problem set 11, it is hard! For final studying: start compiling a list of proof techniques *with simple examples.* The four I gave today (listed above) are your starting point.

Monday April 11: The amazing Professor Stephen Debacker was a guest instructor! The class took Quiz 12 and then worked on a worksheet about using eigenvectors to understand rigid motions in three-space. You proved on the worksheet that the composition of 2 rotations in $\mathbb{R}^3$ is a rotation (around some third axis). The point was that you can find this axis by finding an eigenvector with eigenvalue 1.

Friday April 8: We continued developing our understanding of eigenbases and what it means to *diagonalize* a transformation or matrix using a worksheet. Answers are posted so continue going through and making sure you understand. **Homework:** Read 7.4 and 7.5, do reading webwork, start webwork due wednesday (last one!) and problem set A and B (last ones!). Monday's quiz will draw heavily from the worksheets from MWF this week. In fact at least 2 problems will be exactly the same, including some proof. Answers are posted, so there is no reason to get anything but a perfect score! **My apologies:** I am going to have to miss class Monday and Friday next week, which I really wish were not the case and I am very sorry! On the plus side: I rounded up two of our best and most experienced Math 217 instructors to fill in as surprise guest teachers—one has one numerous teaching awards and the other is also awesome and is currently teaching a strong 217 section (like ours!). You'll still get my quiz and worksheets, I'll still post answers. Final exam is less than two weeks away so please focus just two more weeks! Let's see if our class can have MORE As than NON-As. You know you can do it!

Wednesday April 6: We practiced computing characteristic polynomials, eigenvalues, geometric and algebraic multiplicities, using a worksheet. Please make sure you can do all these techniques! I posted answers for problems A, B and C. If your group did not get through these, please do them yourself and check your answers against mine. Bonus points for finding typos. Webwork! Get started on the problem set! Reread Sections 7.1, 7.2 and 7.3, also read 7.4. You are expected to understand the material in 7.4 starting on page 352—its super important; the beginning of that section is an interesting application but will not be tested on the final.

Monday April 4: We took Quiz 11. If you are not happy with your score, this time, I am asking for an ORAL RETAKE. This can be done after class wednesday OR you can make an appointment. We continued practicing the ideas of eigenvectors and eigenvalues, using a worksheet. We also defined the **λ-eigenspace** of a transformation and saw how to compute it as the kernel of some associated matrix. We defined the **characteristic polynomial** which is important for finding the eigenvalues. Webwork due Wednesday! Continue reading 7.3 and doing reading webwork. Problem set A and B due Friday.

Friday April 1: We started learning about **eigenvectors** and **eigenvalues** (section 7.1) using a worksheet. Webwork! Problem Set! Quiz Monday on stuff related to the worksheet.

Wednesday March 30: Exam Review using a T/F practice sheet. Good luck tonight! See you at 5:50 in LORCH.

Monday March 28: We took Quiz 10. I already posted solutions, so no rewrites for extra credit this time. However, you can earn points back for finding typos and errors in the solutions posted on line to any quizzes and worksheets, as well as the "Change of Basis" Summary and the "Definitions" Summary, both posted on the Section 5 website. We then worked on a review worksheet. Study tips: Do all three exams on Canvas, check your solutions with the posted answers. Reread Chapters 4,5 and 6. Redo worksheets. Write all the definitions on the box on the Definitions Document yourself, then read the document and make sure yours are **exactly** correct. Do all T/F at the back of Chapters 4,5 and 6.

Friday March 25: We discussed the multilinearity property of determinants using a worksheet. This weekend: Please download and start doing the previous year's exams. Also: Read the "Change of Basis" Document on the Website to refresh your memory on Chapter 4. Open the Definitions in Chapters 4, 5 and 6 Document, and try to write out the definitions yourself, then check if you got them correct. Reread the Text Chapters 4, 5 and 6. Do the T/F (with counterexample or justification) at the end of each chapter). Go back through the worksheets and quizzes to make sure you understand your mistakes. Answers on line (bug me if needed). Bonus points on Quiz for finding typos and errors and making good suggestions (unless someone beat you to it). **Monday's Quiz will be on Properties of Determinants, worksheets from this week.**

Wednesday March 23: We talked some about the hard homework set, and I gave some hints. We then discussed properties of determinants using a worksheet.

Monday March 21: We took Quiz 9. I have not posted answers yet in case you want to rewrite for half credit before next class. We studied the **geometric meaning of the determinant** using a worksheet. The punchline is: *an $n \times n$ matrix A gives a map $\mathbb{R}^n \to \mathbb{R}^n$ which maps the unit n-cube in the source to a "parallelopiped" in the target. The* n**-volume** *of this parallelopiped is* **absolute value of the determinant.** You proved this using QR-factorization: write $A = QR$ where Q is orthogonal and R is upper triangular, so $\det A = \det Q \det R = \pm \det R$, which is the product of the diagonal elements of R. These diagonal elements are the lengths of the various heights of the parallelopiped. You'll be reading this this week in 6.3, so pay attention! It should make sense now after you played with it. **Homework:** WEBWORK DUE WED. Problem Set 9 is DIFFICULT. Please get started! read 6.3 and do reading webwork. Exam 2 is next week!

Friday March 18: We started discussing **determinants.** Our definition will be the **Laplace Expansion,** rather than the definition 6.1. But you will not have to formally state the definition on quizzes or tests, rather **know how to compute** the determinant of any $n \times n$ matrix (and of course the important properties of it). Laplace expansion is the best way and works in for any size (square) matrix. We also studied many properties of determinants, most of which follow from the **important theorem:** *If A and B are both $n \times n$, then then* $\det AB = \det A \det B$. In particular: **the determinant respects multiplication of matrices** but **not** addition or scalar multiplication! **Quiz Monday:** Mainly we will cover 5.5. Be sure you know what an inner product is, and how to compute in inner product spaces—for example, finding lengths of vectors or using Gram Schmidt to orthonormalize a basis. Also: be sure you can compute the closest in some subspace to a give vector. For next time: Webwork 9, and read 6.2. Also: get going on Problem Set 9 part A and B.

Wednesday March 16: We continued working on inner product spaces in groups at the white boards, using the same worksheet. Everyone should now have thought about the inner product space of **continuous functions** (on some interval). Here, the inner product is given by **integration:** $\langle f, g\rangle = \int_{-1}^{1} f\, g\, dx$.

You all checked that the axioms of an inner product are satisfied in this context, and then did many computations in this context such as finding magnitudes of vectors (functions), finding the distance between functions, finding orthonormal bases using Gram-Schmidt, etc. The main point of an inner product space is that it is a space where not only can we add and take scalar multiples (ie, a vector space) but *also we can talk about two elements being close to eachother* (the distance between them is small). Actually, we can do much more: we can talk about orthonormal bases, which allows us to easily compute the projection onto some nice subspaces—this will be the **closest element in that nice subspace to our original element.** Please re-read 5.5, especially the second half on the very cool application of these ideas to **approximating arbitrary continuous functions by trig functions** by projecting onto the subspace of trig functions. This is called Fourier analysis and is a huge (super useful) subject you will eventually learn if you major in any math-heavy field. **For Next time:** Finish Problem Set 8 A and B. Read 6.1 on Determinants and do the webwork. Friday we will learn another way to compute the determinant.

Monday March 14: Thanks to Margaret for bringing Moon Pies for π day! We took Quiz 8. If you are dissatisfied with your performance, please download and do over. We then started working on **inner product spaces.** The point here is to introduce something like dot product on any vector space—this is called an inner product. Students started checking, in groups, that certain given sets were inner product spaces. We will continue next time. Please re-read 5.5 and make sure you understand. Also, do Webwork 8 (due Wednesday) and get going on the (pretty hard!) problem set, Part A and B. We will have a shortened class period next time because some researcher from the ed school are coming to observe and talk to you the last 20 minutes of class.

Friday March 11: We worked on some hard proofs (from 5.3) from a worksheet, and the idea behind ”least squares solutions” (5.4) from another. Both are posted. The Quiz Monday on 5.1-5.4 will involve: 1) at least one definition from the BOOK/Worksheets, 2) The proof that orthogonal transformations respect dot product (this is problem H from the worksheet done Friday) and 3). Finding least squares solution. Please try the proof on your own (we did not go over it in class) and practice it. **I will post the solutions to the worksheets** but I am very busy friday so bug me if I forget! This weekend: 1) finish the web homework due Wednesday; 2) Study for Quiz by making sure you know what least squares solutions means AND can do Problem H (proof of the important theorem from Friday’s worksheet (also on Wednesday’s worksheet); 3). Read 5.5 and do the webwork. This involves moving into a more abstract way of thinking, so read thoughtfully. We will start working with inner product spaces (which is an abstractification of the idea of $\mathbb{R}^n$ with the dot product) in 5.5. You need to have read it to be able to get a lot out of class next week.

Wednesday March 9: We took Quiz 7. It is posted so you can redo if you want. We then worked on Orthogonal transformations (5.3). We completed Page 1 of a worksheet (page 2 next time). By **definition,** an orthogonal transformation $\mathbb{R}^n \to \mathbb{R}^n$ is one that preserves the length of every vector. By a theorem (which you will prove next time), an orthonormal transformation also preserves **angles between vectors**. So geometrically, an orthogonal transformation sends every figure to a congruent figure. The (standard) matrix of an orthogonal transformation is an orthogonal matrix, which means its **columns are an orthonormal basis** for $\mathbb{R}^n$. Conversely, if the columns of a square matrix A are orthonormal, then the map given by multiplication by A is orthogonal. Note that the very geometric property of "a linear map preserving lengths and angles" turns out to be the same as the very algebraic property of the corresponding matrix "that its columns are orthonormal." **Assignments:** Finish webwork 7 and problem set 7. Read 5.4 again and (if it is due Friday) do the reading webwork for 5.5. [It may be changed to monday.]

Monday March 7: Welcome back! We went over a worksheet whose purpose was to understand where the QR factorization comes from. It is an application of change of basis. We also practiced Gram-Schmidt. The Quiz was postponed until Wednesday. On Wednesday, you will be asked to find a QR factorization and use Gram-Schmidt. You also need to understand what **projection** means and how it is coming up in G-S. A good way to make sure you understand what is going on is to be sure you understand why the book's description of the R matrix is the same as ours (See Section 5.2). For next time: Read 5.3 and 5.4, do Webwork, prepare for QUIZ and make good progress on Problem Set part A and B.

Friday February 26: We worked on understanding orthonormal projections, using a worksheet on 5.1/5.2. Please go through it and make sure you understand. Over break: read and understand 5.1 and 5.2 well. The **Gram-Schmidt** process for "orthonormalizing" a basis is especially crucial. Finish Webwork 7 (if you are stuck, I can help monday, or ask your friends from class). Read 5.3 and do the reading webwork. Get going on problem set 7, parts A and B. This is a good time to get caught up on things that are not solid. For many of you, that might be the $\mathcal{B}$-matrix and $S_{\mathcal{B}\to A}$ matrix stuff. *Please read my short write-up on the Section 6 website from the link "Change of Coordinates."* It just summarizes everything you need to know on that stuff without proofs (but examples, yes). Also, re-read 4.3 in the book. If you want all the proofs, as the many A-students in our section surely do, the same material in expanded form is in the longer write-up called "Definitions and Main Theorems in Book Order." Making good progress over break will make life more pleasant next week: we will be using all the stuff we have learned in the first half so make sure it is solid. **If you are behind,** contact someone in the class and work together. Make a new friend or strengthen an acquaintance into friendship! MOST IMPORTANT: Relax some over break too! I WILL NOT BE HERE FOR OFFICE HOURS OVER BREAK BUT CAN MEET YOU FRIDAY, SO SEND AN EMAIL IF YOU WANT TO SEE ME.

Wednesday February 24: We took Quiz 6. If you are dissatisfied with your score, you can retake and hand in for up to half credit. We the worked on a worksheet for Section 5.1.

The take away points are these: 1) the **definition** of an orthonormal basis (be sure you state it precisely, in algebraic symbols); 2). the **intuition** of an orthonormal basis (unit vectors that are perpendicular to eachother); 3). the reason we **love orthonormal bases:** there is a specific, easy procedure for finding the coordinates of any vector in an orthonormal basis (make sure you know it!); and 4) It can be tricky to find orthonormal bases for a given subspace of $\mathbb{R}^n$. In 5.2, we will learn a specific technique, called *Gram-Schmidt orthonormalization* for constructing orthonormal bases. If you sweated though some of the worksheet problems today, you will appreciate this technique. **Webwork due Wed! Problem Set A and B due Friday. Also, read 5.2 and do the reading webwork.**

Monday February 22: We worked on linear transformations and vector spaces that are **NOT** just $\mathbb{R}^n$. The things to understand are *coordinates, the matrix of a transformation in a basis* $\mathcal{B}$, and *changing coordinates/basis.* We practiced writing down the $\mathcal{B}$-matrix and change of coordinates matrix $S_{\mathcal{B}\to\mathcal{A}}$. There will be a quiz wednesday on Section 4.3 of the Book. You will will have to find the $\mathcal{B}$ matrix very similar to problem B on the worksheet from today (webwork 6 is also good practice). You will also have to know the comparison theorem "$[T]_{\mathcal{B}} = S^{-1}[T]_{\mathcal{A}}S$, where $S = S_{\mathcal{B}\to\mathcal{A}}$." (Sorry, I can not give you book theorem number for it because someone still has my book!) A summary of what you need to know is provided in a document I wrote called "Summary of Coordinate Change and all That" which is linked from the Section 5 webpage (click on "coordinate change"). A good way to make sure you understand is to read the first 6 pages of that document carefully and slowly, making sure each sentence makes sense, and do the examples on your own then compare with what I wrote to see if you got it right (extra credit for finding typos!). This stuff is the deepest part of Math 217 and hard, so read and re-read both the book and notes! **Homework:** There is a lot of webwork. We will be working on 5.1 and 5.2 next time after the quiz. Reading webwork for those closes wednesday at 9 am. Also, Web homework 6 is due Wednesday. This should help with quiz preparation. You should be making good progress on Problem Set parts A and B too, due Friday! I will be here Wednesday for help with that if you have your questions ready.

Friday February 19: We took Group Quizzes. There were two. One (Quiz 5A) had basic proofs with similar matrices. The other (Quiz 5B) was a neat trick with matrix multiplication and Fibonacci sequences. Everyone should make sure they can completely understand Quiz 5A (don't stress if you didn't finish 5B, we will get to this later in Chapter 7). We then started thinking about **vector spaces that are not** $\mathbb{R}^n$. We had a worksheet to work on in groups. We will continue next time with the same groups. This weekend: Webwork 6! Now is a good time to re-read and be strong on all of Chapter 4 in the book. Reading, and re-reading the book, slowly, and trying each example yourself before moving on, is one of the ways to learn how to "do technical work" like math (or stats, econ, physics, engineering...) and one of the life-skills you are supposed to learning in Math 217. The students who are consistently reading and re-reading the book/notes scored higher than those who don't. Get started also on the problem set. **Next week, Quiz will be Wednesday instead of Monday.**

Wednesday February 17: Exam review. See you TONIGHT at 5:50 pm in Lorch 140. Exam starts at 6 pm SHARP.

Monday February 15: We took Quiz 5. We worked some more on the $\mathcal{B}$-matrix stuff using a Worksheet. We did not finish the worksheet...some of it is on material not on the Exam, so don't stress. Problems A and B (on the posted version, note that order is slightly different than the distributed version) are from the material which could be on the exam. STUDY: Do all three practice exams, check your work with solutions provided on canvas. We will discuss them next time as needed. If you have done this already, do all the T/F questions in Chapter 1,2 and 3 (at the end of each chapter). Also: study all the summaries at the end of each section in the book and the examples in the text in chapter 1,2, and 3. Make sure your definitions are memorized (see the definitions document).

Friday February 12: We did a practice quiz to practice some proofs. Then we worked on a worksheet on coordinates and $\mathcal{B}$-matrices. We only got through the first page (Problem A). **Homework:** Finish the webwork! This is important for your exam practice in addition to your webwork grade. Three exams are posted: please do all three. Go in reverse chronological order, as we will start with Fall 2015 to review in class. Try to do them *without looking at the solutions.* Quiz Monday will take at least one problem DIRECTLY from one of these exams. Other study tips: Do all the T/F questions at the end of Chapters 1, 2 and 3. Make sure you can justify your answer by giving a brief proof or counterexample. Read the Definitions Document and make sure you can state every definition carefully (except "vector space" as it is too long to be on the exam) by writing it yourself without looking. **EXTRA CREDIT OPPORTUNITY:** Do the Rank-Nullity worksheet (11 questions, handed out today) and turn in next time before class. Explain all your answers...I will replace your worst quiz grade with this one.

Wednesday February 10: We went over how to find the $\mathcal{B}$ matrix of a linear transformation and what it means. Homework: Finish Problem Set Parts A and B. We are wrapping up Chapter 3. The textbook has many good summaries at the end of every section—study these! Finish Web homework set 5. Especially Reread 3.4 in the book, paying attention to "B-matrix" and "change of basis matrix" We will talk more about change of basis matrix friday). Also the "Change of Coordinates and All That" document on my websiet s a good summary of 3.4 which might help. Also, for better/deeper understanding (needed for doing proofs), please read the "Definitions and Main Theorems in Book Order" Document on the webpage.

Monday February 8: We took Quiz 4. As usual, if you are not satisfied with your performance, turn in it for regrading by the beginning of class next time. We then discussed **coordinates.** The point is that if we have a basis $\mathcal{B}$ for a d-dimensional vector space V, then we have a way of *identifying* V with $\mathbb{R}^d$. We simply match up each vector $\vec{v}$ with its *coordinate column* $[\vec{v}]_{\mathcal{B}}$ of $\mathcal{B}$-coordinates. This identification is an *isomorphism of vector spaces.* We also discussed some standard proof techniques:

(1) To show something is **unique**, assume there are two of them, and then do some math to show they must be equal.

(2) To show vectors $\vec{v}_1, \ldots, \vec{v}_d$ are linearly independent, assume there is a relation $c_1\vec{v}_1 + \cdots + c_d\vec{v}_d = 0$, and then do some math to show each $c_i = 0$.
(3) To show a set of vectors $\{\vec{v}_1, \ldots, \vec{v}_d\}$ is a basis for V, show separately that 1) the set spans V and 2) the set is linearly independent.
(4) OR, as a very useful shortcut for showing a set of vectors $\{\vec{v}_1, \ldots, \vec{v}_d\}$ is a basis for V, **if we know the space is d-dimensional and we have d vectors in the set,** then we only have to check EITHER $\{\vec{v}_1, \ldots, \vec{v}_d\}$ spans V OR $\{\vec{v}_1, \ldots, \vec{v}_d\}$ is linearly independent. (Theorem 3.3.4 in the book). That is, if they span, they are automatically linearly independent (if you have the right number of them) and vice versa! This only works for a finite dimensional space where we already know the dimension is equal to the number of vectors in the set we want to show is a basis!

Homework: Read the "Definitions/Theorems in Book Order" Document in its entirety. Reread 3.4 in the book, and try to understand the $\mathcal{B}$-matrix. Read more about $\mathcal{B}$-matrices in the Sections 5 and 6 of the document called "Definitions and Theorems" on the webpage. Work on the Webwork....despite the extension, please don't wait until Saturday because you will get behind. Make progress on the Problem Set. Start reading Chapter 4 (this will be mostly review for our section) and keep up with the reading webwork on it.

Friday February 5: We continued working on the idea of a BASIS. This is an important and deep idea. Please finish the worksheet and make sure you understand everything on it. The QUIZ MONDAY will take at least one problem from this worksheet, including perhaps one of the two proofs at the bottom of the sheet. It will also ask you to do at least proof from chapter 3 of the "Definitions in Book Order Document" on the Section 5 webpage. This is developing all the time....in particular, I will write the proofs of these two Theorems into it this weekend. So read it! Please do Webwork this weekend, and please start on Homework set 5. In addition, read Section 3.4 and the corresponding section in the Definitions Document about Coordinates. This is the hardest part of 217 so far, so read and reread the idea of "coordinates" and "changing coordinates/basis". Exam 1 is in less than 2 weeks and will be pretty heavy on the basis and coordinates stuff!

Wednesday February 3: We took Quiz 3B and went over it, to practice some proofs. We then discussed the words **span, linearly independent, basis,** and **dimension.** This is the hardest topic yet, and super important. Please memorize all definitions exactly as in the definitions document. Linearly independent is especially tricky, and you should make sure you know the correct definition in the document! We started on Worksheet 3.3 (posted) and will continue next time. Please re-read Section 3.2 and 3.3 of the book AND the DEFINITIONS DOCUMENT, which is updated every day. You know what's due today at midnight and friday... if not, check CANVAS AND the WEBWORK PAGE.

Monday February 1: We took Quiz 3. If you are not satisfied with how it went, you can download Quiz 3 and retake it. If you turn it in before class next time, I will average the two scores.

We then discussed the concepts of **kernel, image, subspace and** *spanning set* of a subspace, using a worksheet. These notions are from 3.1 and 3.2. Please make sure you finish the worksheet if you did not get to it in class. Answers are posted, so you can check your work. For next time: READ 3.3 and complete reading webwork as well as the DEFINITIONS DOCUMENT (which has been updated as of today). I will be updating this as we go along. As you might notice, it is still a ”work in progress” beyond Section 3.3. I will try to keep up with your pace in reading the textbook, so you can refer to this on the side for the more general material. Extra credit for finding typos and making suggestions! I want your feedback! Also: webwork is due Wednesday—this is a harder section! Please give yourself time for it! Also, get started on Problem Set 4, parts A and B.

Friday January 29: We discussed **vector spaces.** This is not in the book so *please read the document called ”Definitions” on the Section 5 Webpage.* We did a worksheet on vector spaces. Examples were presented at the board by Mark Spencer, Katie Matton, and Suki Dasher. It is very important, when you are doing the reading from the book in Chapter 3, that you think about the case of general vector spaces as you learn new words such as ”span,” kernel,” ”image,” ”subspace,” ”linearly independent,” ”basis” and ”coordinates.” This weekend: do Web Homework 4 (closes Wednesday but do it this weekend so you are ready for the quiz!). Also read and do the reading webwork for 3.2. There is a QUIZ Monday. The quiz will be on definitions and proofs related to the notion of **vector spaces** and their linear transformations. Be prepared by having read and digested the ”Definitions” document on the website and friday’s worksheet.

Wednesday January 27: We took and went over Quiz 2A (posted on website), without recording grades. If you did poorly, please study (learn what rank is, etc) as you will soon be tested (quiz in class and/or definitely Exam 1) on this material ”for real”. We then practiced some inductive proofs. I wrote out proofs for all the problems on the website: please make sure you understand. We then started talking about **vector spaces,** using a worksheet. This material is not in the book (some of it will be later in Chapter 4), so please look at the DEFINITIONS on the worksheet and/or in the list of definitions published on the website. We will continue with this next time, so please bring the worksheet again. We will also start talking about 3.1. Read and do the reading webwork for 3.1 before 8 am friday. ALSO: problem Set 3 is due, parts A and B friday in class.

Monday January 25: We took Quiz 2. If you are not satisfied with your performance, you can retake it (download from website) are turn in by 11:40 am wednesday. We then discussed surjective, injective and invertible maps in the context of *linear transformations.* We computed the image of a linear transformation T_A from $\mathbb{R}^3$ to $\mathbb{R}^3$ by thinking about solving a system of linear equations $A\vec{x} = \vec{y}$ and identifying the values of $\vec{y}$ for which the system is *consistent.* This was a hard concept for many students....please read the solutions to the worksheet and make sure you understand! For the problem set, you need to know what a **vector space** is. I have uploaded a list of Math 217 Definitions where this is defined. Please take a look! Webwork due Wednesday! Problem Set due Friday!

Friday January 22: We discussed 2.3. The main points: 1) a composition of linear transformations is a linear transformation. You should be able to prove this directly from the definition of linear transformation. 2) MOREOVER: If T and S are linear transformations with matrices A and B, respectively, then $T \circ S$ has matrix AB. Be very careful in thinking about the source and target of compositions. The notation $T \circ S$ means the transformation S acts first, then T, so

$$T \circ S(\vec{x}) = T(S(\vec{x})) = T(B\vec{x}) = A(B\vec{x})) = (AB)\vec{x}.$$

We used a worksheet to understand all this, posted on the website. Also: we practiced ideas like "identity linear transformation" and block matrix multiplication. FINISH the wednesday WEBWORK this weekend, read 2.4 and do the reading work, PLUS get started on the problem set (A and B). QUIZ MONDAY will cover problems on worksheets from class last week. For worksheets, try to make sure you know what the main point is of each.

Wednesday January 20: We discussed the geometric definition of the determinant: For an 2×2 matrix, the absolute value of the determinant tells us the "scale factor" for areas (see worksheet on determinants). We also investigated projections and reflections in the plane, saw that they are linear, and then derived the corresponding matrices (see worksheet on projections and reflections). Solutions for both worksheets are posted....go through these! Note: we did not get to problem B on "reflections" on the second worksheet. We will not be returning to this next time because we must move on to 2.3 but serious/interested/curious/ambitious/A students should try it first on their own, then compare their results to the answers on the worksheet. It is also explained in the book (Section 2.2). For next time: you should read 2.3 and do the reading webwork. Of course, Problem set 2 (part A and B) is also due. The problem set is HARD! Typically many 217 students gather in the math atrium Thursday nights to discuss...I'll try to swing by there in the range of 6:30 —8 pm and see if you need some hints. Next time: START EARLIER!

Friday January 15: We took Quiz 1A and went over it. There are two main points. The first is that you **must memorize** the following definition of **Linear Transformation:** *A linear transformation* $\mathbb{R}^n \xrightarrow{T} \mathbb{R}^m$ *is a mapping such that* $T(\vec{x} + \vec{y}) = T(\vec{x}) + T(\vec{y})$ *for all vectors* $x, y \in \mathbb{R}^n$ *AND* $T(a\vec{x}) = aT(\vec{x})$ *for all vectors* $\vec{x}$ *in the source and all scalars* a. The second is that *it every linear transformation* $\mathbb{R}^n \xrightarrow{T} \mathbb{R}^m$ *can be described as a* **matrix multiplication.** We discussed how to find the matrix A of a linear transformation—the point is you can find the columns by tracking the images of the standard unit vectors. **Homework:** Since Monday is MLK day, please come prepared for a quiz on 2.1 and 2.2 Wednesday instead on 2.1 and 2.2. These are both fundamental sections, so you should plan to read them more than once. The book's definition of linear transformation is different from ours, so please make sure you see why they are equivalent. Do the web homework due Wedensday BEFORE class as a way to prepare for a quiz. I may decide to skip the quiz, but be prepared. Also, read 2.3 and do the reading webwork.

Wednesday January 13: We discussed two big topics: *linear combinations* and *linear transformations.* There is a worksheet on each, which I have updated with answers. Please make sure you understand, and ask questions if you don't. Especially the idea of a *linear transformation* is difficult. Our definition is somewhat different from the book's but they are equivalent. You must know **both** definitions and understand why they are equivalent. Webwork is due at 11:59 tonight! Also, read 2.2 and do that webwork for Friday. Of course, problem set A and B are both due Friday in class as well. Office hours will be Wednesdays 1-4.

Monday January 11: We took Quiz 1. We then went over it and reviewed the idea of the **rank** of a system of linear equations and what that has to do with the dimension of the set of solutions. **Vocabulary:** rank, augmented matrix, pivots, free/independent variables, dependent variables. We also practiced some proofs, like showing "The square of an odd integer is odd." Challenge problem: *Prove that* $\sqrt{2}$ *is irrational."* For Wednesday: Read 2.1 and do the reading webwork. THREE LONG webwork assignments are also due. Be sure to get going on the problem set DUE FRIDAY as well.

Friday January 8: Sorry I had to miss class. Diego Ayala subbed. We covered 1.2 on solving linear systems and Gaussian Elimination. I have posted the answers to the worksheet. Please go over them! Quiz Monday on solving systems of equations and row reduced echelon form. The webwork assignment that closes wednesday on 1.1 and 1.2 (not the reading but the main assignment) is a good way to practice for the quiz.

Wednesday January 6: Material Covered from book: Section 1.1. We went over course expectations, and worked on the first worksheet (about systems of linear equations). It is posted on the Section 5 website; be sure you understand.

Homework: Read Sections 1.1 and 1.2 in the book by Friday and complete the Reading Webwork by 8 am Friday. Read the two handouts, *Joy of Sets* and *Mathematical Hygiene*, posted on the website. Get started on the webwork—there is a ton due Wednesday! Problem Set 1, parts A and B, is due NEXT friday. It should already be on Canvas, you should take a look now and plan to make progress this weekend.

Issues: If you enrolled late, you may not be in the webwork system. The people in charge are working on this. Let me know, but don't stress as we will not hold it against you if you miss a deadline in this case. If you are still on the waitlist, go talk to staff in the Math office in East Hall. I don't handle the overrides.

Next Time: We will cover 1.2. Be sure to read it in advance. Sorry, but there will be a sub. Quiz Monday on 1.1 and 1.2.