

# Math 295: Logic Practice

1. Express each of the following statements in a formal mathematical way by using “if, then” statements or/and quantifiers (“for all ” and “there exists”). Use  $S$  for the set of all students, and  $M(x)$  for the statement “ $x$  likes math,” and  $P(x)$  for the statement “ $x$  likes physics.” Introduce new symbols as needed, and be sure to define them.

1. All students who like mathematics also like physics.
2. Some students like mathematics, and some students like physics.
3. Some students who like mathematics also like physics.
4. Only students who like mathematics also like physics.
5. There are some students who like Oscar but do not like George.
6. No student who likes mathematics also likes Oscar.

2. Translate each of the following formal statements about all animals into English. The symbols are defined as follows:  $C(x)$  means  $x$  is a cat;  $D(x)$  means  $x$  is a dog;  $T(x)$  means  $x$  has a tail;  $B(x, y)$  means  $x$  bites  $y$ ;  $L(x, y)$  means  $x$  likes  $y$ .

1.  $\exists x$  such that  $C(x)$  and  $\neg T(x)$ .
2.  $\forall x, D(x)$  implies  $T(x)$ .
3.  $\forall x$  satisfying  $C(x)$ ,  $\exists y$  such that  $D(y)$  and  $B(x, y)$ .
4.  $\forall x, \exists y$  satisfying  $D(y)$  such that if  $L(x, y)$ , then  $C(x)$ .
5.  $\exists x$  satisfying  $C(x)$  such that  $\forall y$  satisfying  $D(y)$ , we have  $B(x, y)$ .
6.  $\forall x$  and  $\forall y$ , if  $D(y)$  implies  $B(x, y)$ , then  $C(x)$ .
7.  $\forall x$ , if both  $T(x)$  and  $B(x, x)$ , then  $D(x)$ .

3. For every sentence in 1 and 2, write the contradiction (or negation). That is, write the sentence that expresses precisely the fact that the given sentence is FALSE. Compare your answers in math and english syntax.