Math 295: Practice with concepts

1. Suppose $\star$ is a commutative binary operation on a set S and that there exists an element $e \in S$ such that $e \star a = a$ for all $a \in S$. Is there a $\star$-identity? What if $\star$ was not commutative?

2. True or False (justify): The usual operation of multiplication on the set $\{-1, 1\}$ is commutative, associative, binary operation with identity and an inverse for every element.

3. True or False (justify): There exists a proper inductive subset of $\mathbb{N}$.

4. True or false (justify): There exists a well-ordered subset of $\mathbb{R}$ which contains only irrational numbers.

5. True or false (justify): The rational numbers are a complete ordered field.

6. True or false (justify): The multiplicative inverse of a non-zero element of a field is a unique.

7. True or false (justify): Every field admits an order structure, though not always in a natural way.

8. True or false (justify): There is at most one way to order a field.

9. True or false (justify): There is an ordered field in which the multiplicative identity is non-positive.

10. True or false (justify): The set of all even integers is well-ordered.

11. Let $P(S)$ be the set of all subsets of a fixed set S. Is the operation of $\cup$ (union) a binary operation on $P(S)$. Is it commutative? Associative? Have an identity? Which elements have inverses?

12. Again, let $P(S)$ be the set of all subsets of a fixed set S. Is the operation of $\cap$ (intersection). Is this a binary operation on $P(S)$. Is it commutative? Associative? Have an identity? Which elements have inverses?

13. Is there any form of the distributive property that holds for the binary operations $\cap$ and $\cup$ on $P(S)$?

14. True or false (justify): Every ordered field contains an inductive set.

15. Give an example of a bounded above subset of $\mathbb{Q}$ which does not admit a supremum (or explain why such can't exist).

16. Give an example of a bounded below subset of $\mathbb{R}$ which does not admit an infimum (or explain why such can't exist).

17. True or false (justify): If A is a proper subset of a set B, there can not be a bijection $A \to B$.

18. True or false (justify): The function $f : \mathbb{R} \to \mathbb{R}_{\geq 0}$ sending $x \mapsto e^x$ is a bijection.

19. True or false (justify): A function if bijective if and only if it has an inverse function.

20. True or false (justify): If $f : A \to B$ is a function, and $g : B \to A$ is another function such that $f \circ g = id_B$, then f is bijective.

21.True or false (justify): If $f : A \to B$ is a function, and $g : B \to A$ is another function such that $f \circ g = id_B$, then f is injective.

22. True or false (justify): If $f : A \to B$ is a function, and $g : B \to A$ is another function such that $f \circ g = id_B$, then g is injective.

23. True or false (justify): Every element in a field has a multiplicative inverse.

24. True or false (justify): The set of all even integers is countable.

25. True or false (justify): If A is a proper subset of B, then A and B can not have the same cardinality.

26. True or false (justify): Same as 25, with B a finite set.