

## Math 351 – Winter 2006 – Team Homework 7 – Continuous Functions

### Problem 1

Determine whether the following functions on given sets are discontinuous, continuous or uniformly continuous. (If a function is both continuous and uniformly continuous on a given set, mark only “uniformly continuous”.) Please, clearly mark your answer in each problem.

1.  $f(x) = 2^{x^2}$ ,  $x \in (-5, 5)$ .

DISCONTINUOUS                      CONTINUOUS                      UNIFORMLY CONTINUOUS

2.  $f(x) = 2^{x^2}$ ,  $x \in \mathbb{R}$ .

DISCONTINUOUS                      CONTINUOUS                      UNIFORMLY CONTINUOUS

3.  $f(x) = x \sin\left(\frac{1}{x}\right)$ ,  $x \in (-1, 1)$ .

DISCONTINUOUS                      CONTINUOUS                      UNIFORMLY CONTINUOUS

4.  $f(x) = x^2 \sin\left(\frac{1}{x}\right)$ ,  $x \in \mathbb{R}$ .

DISCONTINUOUS                      CONTINUOUS                      UNIFORMLY CONTINUOUS

5.  $f(x) = \tan x$ ,  $x \in (-\pi/2, \pi/2)$ .

DISCONTINUOUS                      CONTINUOUS                      UNIFORMLY CONTINUOUS

6.  $f(x) = \tan^2 x$ ,  $x \in \mathbb{R}$ .

DISCONTINUOUS                      CONTINUOUS                      UNIFORMLY CONTINUOUS

7.  $f(x) = \frac{1}{x} \sin\left(\frac{1}{x^2}\right)$ ,  $x \in (-0.1, 0.1)$ .

DISCONTINUOUS                      CONTINUOUS                      UNIFORMLY CONTINUOUS

8.  $f(x) = \frac{1}{x} \sin\left(\frac{1}{x^2}\right)$ ,  $x \in (0.1, 1500)$ .

DISCONTINUOUS                      CONTINUOUS                      UNIFORMLY CONTINUOUS

9.  $f(x) = \sin\left(\frac{1}{x}\right)$  for  $x \neq 0$  and  $f(x) = 0$  for  $x = 0$  on a set  $x \in \mathbb{R}$ .
- DISCONTINUOUS                      CONTINUOUS                      UNIFORMLY CONTINUOUS
10.  $f(x) = \sin\left(\frac{1}{x}\right)$ ,  $x \in (-\infty, 0) \cup (0, \infty)$ .
- DISCONTINUOUS                      CONTINUOUS                      UNIFORMLY CONTINUOUS
11.  $f(x) = x^{12} \sin(3x + 2) - e^x \cos(2x)$ ,  $x \in [-1, 4]$ .
- DISCONTINUOUS                      CONTINUOUS                      UNIFORMLY CONTINUOUS
12.  $f(x) = \frac{x+2}{x+6}$ ,  $x \in [-1, 4]$ .
- DISCONTINUOUS                      CONTINUOUS                      UNIFORMLY CONTINUOUS
13.  $f(x) = \frac{x+2}{x+6}$ ,  $x \in (-1, 4)$ .
- DISCONTINUOUS                      CONTINUOUS                      UNIFORMLY CONTINUOUS
14.  $f(x) = \frac{x+2}{x+6}$ ,  $x \in (-6, 4]$ .
- DISCONTINUOUS                      CONTINUOUS                      UNIFORMLY CONTINUOUS
15.  $f(x) = x^2 \operatorname{sgn} x$ , where  $\operatorname{sgn} x = 1$  for  $x > 0$ ,  $\operatorname{sgn} x = 0$  for  $x = 0$  and  $\operatorname{sgn} x = -1$  for  $x < 0$  on a set  $x \in \mathbb{R}$ .
- DISCONTINUOUS                      CONTINUOUS                      UNIFORMLY CONTINUOUS

**Problem 2**

In each part determine whether functions with the following properties exist. If so, provide an example of such a function. If not, you don't need to give a proof. Please, clearly mark your answer in each problem. Note that last 5 problems marked **HARD** are not required but you can earn an extra credit for solving them.

1.  $f(x)$  is defined on  $\mathbb{R}$  and is unbounded on each interval.

DOES NOT EXIST                      EXISTS, EXAMPLE:

2.  $f(x)$  is defined on  $\mathbb{R}$  and is continuous everywhere except at  $\{0, 1, 2, 3, \pi\}$ .

DOES NOT EXIST

EXISTS, EXAMPLE:

3.  $f(x)$  is defined on  $\mathbb{R}$  and is discontinuous everywhere except at  $\{0, 1, 2, 3, \pi\}$ .

DOES NOT EXIST

EXISTS, EXAMPLE:

4.  $f(x)$  is defined on  $\mathbb{R}$  and is discontinuous everywhere except on an interval  $x \in (0, 1)$ .

DOES NOT EXIST

EXISTS, EXAMPLE:

5.  $f(x)$  is defined on  $\mathbb{R}$  and is discontinuous everywhere except on an interval  $x \in [0, 1]$ .

DOES NOT EXIST

EXISTS, EXAMPLE:

6.  $f(x)$  is defined on  $\mathbb{R}$  and is continuous everywhere except on an interval  $x \in (0, 1)$ .

DOES NOT EXIST

EXISTS, EXAMPLE:

7.  $f(x)$  is defined on  $\mathbb{R}$  and is continuous everywhere except on an interval  $x \in [0, 1]$ .

DOES NOT EXIST

EXISTS, EXAMPLE:

8.  $f(x)$  is defined on  $\mathbb{R}$ , is monotone and bounded and continuous except discontinuous at every integer  $n$ .

DOES NOT EXIST

EXISTS, EXAMPLE:

9.  $f(x)$  is defined on  $\mathbb{R}$ , is continuous, invertible with a discontinuous inverse.

DOES NOT EXIST                  EXISTS, EXAMPLE:

10.  $f(x)$  is defined on  $[0, 5]$ , is continuous,  $f(0) = 3$ ,  $f(5) = -2$  and  $f(x) = 0$  has exactly 4 different solutions in  $[0, 5]$ .

DOES NOT EXIST                  EXISTS, EXAMPLE:

11. **EXTRA HARD**  $f(x)$  is defined on  $\mathbb{R}$ , is discontinuous at every irrational number and continuous at every rational number.

DOES NOT EXIST                  EXISTS, EXAMPLE:

12. **HARD**  $f(x)$  is defined on  $\mathbb{R}$ , is bounded and is discontinuous at every real number.

DOES NOT EXIST                  EXISTS, EXAMPLE:

13. **HARD**  $f(x)$  is defined on  $\mathbb{R}$ , is continuous at every irrational number and discontinuous at every rational number.

DOES NOT EXIST                  EXISTS, EXAMPLE:

14. **HARD**  $f(x)$  is defined on  $\mathbb{R}$ , is continuous at each irrational number, is discontinuous otherwise but is unbounded on each interval.

DOES NOT EXIST                  EXISTS, EXAMPLE:

15. **EXTRA HARD**  $f(x)$  is defined on  $\mathbb{R}$ , is monotone and bounded and discontinuous at every rational number.

DOES NOT EXIST                  EXISTS, EXAMPLE: