Stability of Nonlinear Waves

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The goal of this course is to study stability of traveling waves and shock waves that are important solutions of partial differential equations modeling various phenomena in Physics, Chemistry, or Biology. Although some of these structures are only transient, many of them are robust and persist for long time. In mathematical terms one needs to study their stability. Stability studies are now ubiquitous in the field of applied mathematics both because of their relevance to understand phenomena and availability of powerful techniques developed over the last 10-15 years.

In the course we are going to survey these techniques along with recent advances in the field of stability of nonlinear waves. The methods used are often originating in the theory of dynamical systems, i.e. ordinary differential equations. Among the topics covered you may find: Linear, nonlinear, and energetic stability, Evans function, Gap Lemma, and Krein signature. The emphasis will be on geometric interpretations of concepts.

Prerequisites: Background in Ordinary and Partial Differential Equations (some of Math 404, Math 454, Math 656, Math 657).

The course workload will be light with reading assignments and a small project.

Literature:

1. Sandstede B – Stability of Travelling Waves, Handbook of Dynamical Systems Vol. II, 2002.
2. Grillakis M, Shatah J, Strauss W – Stability theory of solitary waves in the presence of symmetry I-II, J. Funct. Anal 74, 160-197 (1987); 94, 308-348 (1990).
3. Maddocks J – Restricted quadratic forms and their application, SIAM J. Math. Anal. 16, 47-68 (1985) and 19, 1256-1257 (1988).
4. Alexander J, Gardner RA , Jones CKRT – A topological invariant arising in the stability anlysis of traveling waves, J. Reine Angew. Math, 410, 167-212 (1990).
5. Pego R, Weinstein M – Eigenvalues, and instabilities of solitary waves, Phil Trans. R. Soc. Lond. A, 340, 47-94 (1992).
6. Gardner RA, Zumbrun K – The gap lemma and geometric criteria for instability of viscous shock profiles, Comm. Pure. Appl. Math. 5, 797-855 (1998).
7. Kapitula T – Stability analysis of pulses via the Evans function. Lecture Notes in Phys., 661, Springer, Berlin, 407–428, 2005.
8. Kapitula T, Kevrekidis PG, Sandstede B, Counting eigenvalues via the Krein signature, Physica D 195, 263–282 (2004), 201, 199–201 (2005).
9. Pelinovsky DE, Intertia law for spectral stability of solitary waves in coupled NLS equations, Proc. R. Soc. Lond. Ser. A 461, 783–812 (2005).