1. Show that \( \int_a^b (f(x) + g(x)) \, dx = \int_a^b f(x) \, dx + \int_a^b g(x) \, dx \) using Riemann sums.

2. Show that \( \sum_{i=1}^n i^3 = \left( \frac{n(n+1)}{2} \right)^2 \) using two different methods as indicated below.

   method 1 Use a telescoping sum as in class.

   method 2 Consider a square where each side has segments of length 1, 2, \ldots, \( n \), so the length of each side is 1 + 2 + \ldots + n = \left( \frac{n(n+1)}{2} \right) \), and hence the area of the square is \( A = \left( \frac{n(n+1)}{2} \right)^2 \). Now consider subregions of area \( a_1, a_2, \ldots, a_n \), where \( a_1 = 1 \) is the area of a unit square, and \( a_i \) for \( i = 2, \ldots, n \) is the area of the L-shaped subregion as shown in the figure. Show that \( a_i = i^3 \) and hence the area of the square is also equal to \( A = a_1 + a_2 + a_3 + \ldots + a_n = \sum_{i=1}^n i^3 \).

3. Evaluate \( \int_0^1 x^3 \, dx \) two ways.
   a) Riemann sums  b) FTC

4. Evaluate \( \int_a^b x \, dx \) by Riemann sums.

5. a) Express \( \lim_{n \to \infty} \frac{1}{n} \sum_{i=1}^n \frac{1}{1+(i/n)^2} \) as an integral. b) Find the derivative of \( f(x) = \int_0^x \sqrt{1 + t^3} \, dt \).

6. Evaluate by any method. a) \( \lim_{n \to \infty} \frac{1}{n} \left( \sqrt{1 + \frac{1}{n}} + \sqrt{2 - \frac{1}{n}} + \ldots + \sqrt{n} \right) \)  b) \( \int_0^1 \frac{dx}{\sqrt{x}} \)

7. A steel rod of length \( L \) cm has variable cross-sectional area \( A(x) \) cm\(^2\), where \( x \) is measured in centimeters from one end of the rod. The rod has uniform mass density \( \rho \) g/cm\(^3\). (a) Derive an expression for the total mass \( M \) of the rod. (hint: think of slices) (b) Compute the total mass for the case \( L = 25 \) cm, \( A(x) = 9 + 2\sqrt{x} \) cm\(^2\), \( \rho = 8 \) g/cm\(^3\). Express \( M \) in kilograms (kg).

8. a) Derive the formula for the sum of a finite geometric series.

\[
\sum_{i=0}^{n} r^i = 1 + r + r^2 + \ldots + r^n = \frac{1 - r^{n+1}}{1 - r}, \quad \text{if } r \neq 1.
\]

(hint: check the formula for \( n = 0, 1, 2 \), and then show it is true in general.)

b) A student obtains a $1,000 loan and repays 50% of the balance each year, i.e. $500 is repaid in year 1, $250 is repaid in year 2, and so on. Express the total amount repaid after 10 years as a series and evaluate it using part (a).

c) Evaluate \( \int_0^1 e^x \, dx \) by Riemann sums. (this completes problem 5c from hw1)

d) What happens to the formula in part (a) in the limit \( r \to 1 \)?

9. Consider the integral \( I = \int_0^1 e^{-x} \, dx = 1 - e^{-1} = 0.63212056 \). Let \( R_n, M_n \) be the right-hand and midpoint Riemann sums with \( n \) intervals. Construct a table as follows (use a calculator), column 1: \( n \) (take \( n = 1, 2, 4 \)); column 2: \( \Delta x \); column 3: \( R_n \); column 4: \( |I - R_n| \); column 5: \( M_n \); column 6: \( |I - M_n| \). For a given value of \( n \), which method is more accurate? When \( \Delta x \) decreases by 1/2, by what factor does the error decrease for each method?

10. Sketch the graphs of \( y_1 = e^x, y_2 = x, y_3 = \ln x \) in one plot. Note that \( y_1, y_2 \) are defined for \( -\infty < x < \infty \), and \( y_3 \) is defined for \( 0 < x < \infty \).

announcement The Science Learning Center offers study groups for Math 156 students. Online registration begins on Wednesday Sept 11 at 12pm via www.lsa.umich.edu/slc and groups start meeting on Sunday Sept 15. If the group you want is filled, please join the waitlist and another group may be opened.