Math 156 Applied Honors Calculus II Fall 2019

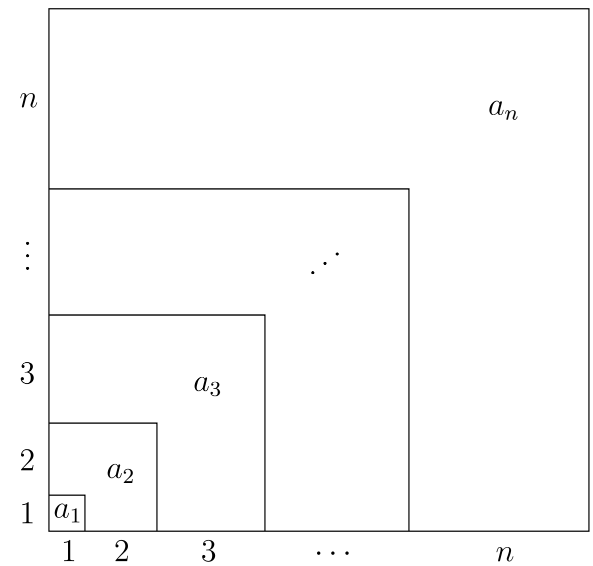
hw2 , due: Tuesday, September 17

1. Show that $\int_a^b (f(x)+g(x))dx = \int_a^b f(x)dx + \int_a^b g(x)dx$ using Riemann sums.

2. Show that $\sum\limits_{i=1}^{n} i^3 = \left(\frac{n(n+1)}{2}\right)^2$ using two different methods as indicated below.

<u>method 1</u> Use a telescoping sum as in class.

<u>method 2</u> Consider a square where each side has segments of length $1, 2, \ldots, n$, so the length of each side is $1+2+\cdots+n = \frac{n(n+1)}{2}$, and hence the area of the square is $A = \left(\frac{n(n+1)}{2}\right)^2$. Now consider subregions of area $a_1, a_2, \ldots, a_n$, where $a_1 = 1$ is the area of a unit square, and a_i for $i = 2, \ldots, n$ is the area of the L-shaped subregion as shown in the figure. Show that $a_i = i^3$ and hence the area of the square is also equal to $A = a_1 + a_2 + a_3 + \cdots a_n = \sum\limits_{i=1}^{n} i^3$.



3. Evaluate $\int_0^1 x^3 dx$ two ways.

a) Riemann sums b) FTC

4. Evaluate $\int_a^b x dx$ by Riemann sums.

5. a) Express $\lim\limits_{n\to\infty} \frac{1}{n} \sum\limits_{i=1}^{n} \frac{1}{1+(i/n)^2}$ as an integral. b) Find the derivative of $f(x) = \int_0^{x^2} \sqrt{1+t^3}\, dt$.

6. Evaluate by any method. a) $\lim\limits_{n\to\infty} \frac{1}{n}\left(\sqrt{\frac{1}{n}} + \sqrt{\frac{2}{n}} + \sqrt{\frac{3}{n}} + \cdots + \sqrt{\frac{n}{n}}\right)$ b) $\int_1^4 \frac{dx}{\sqrt{x}}$

7. A steel rod of length L cm has variable cross-sectional area $A(x)$ cm^2, where x is measured in centimeters from one end of the rod. The rod has uniform mass density ρ g/cm^3. (a) Derive an expression for the total mass M of the rod. (hint: think of slices) (b) Compute the total mass for the case $L = 25$ cm, $A(x) = 9 + 2\sqrt{x}$ cm^2, $\rho = 8$ g/cm^3. Express M in kilograms (kg).

8. a) Derive the formula for the sum of a <u>finite geometric series</u>,

$$\sum_{i=0}^{n} r^i = 1 + r + r^2 + \cdots + r^n = \frac{1-r^{n+1}}{1-r}\ , \text{ if } r \neq 1.$$

(hint: check the formula for $n = 0, 1, 2$, and then show it is true in general.)

b) A student obtains a \$1,000 loan and repays 50% of the balance each year, i.e. \$500 is repaid in year 1, \$250 is repaid in year 2, and so on. Express the total amount repaid after 10 years as a series and evaluate it using part (a).

c) Evaluate $\int_0^1 e^x dx$ by Riemann sums. (this completes problem 5c from hw1)

d) What happens to the formula in part (a) in the limit $r \to 1$?

9. Consider the integral $I = \int_0^1 e^{-x} dx = 1 - e^{-1} = 0.63212056$. Let R_n, M_n be the right-hand and midpoint Riemann sums with n intervals. Construct a table as follows (use a calculator). column 1: n (take $n = 1, 2, 4$); column 2: Δx; column 3: R_n; column 4: $|I - R_n|$; column 5: M_n; column 6: $|I - M_n|$. For a given value of n, which method is more accurate? When Δx decreases by 1/2, by what factor does the error decrease for each method?

10. Sketch the graphs of $y_1 = e^x, y_2 = x, y_3 = \ln x$ in one plot. Note that y_1, y_2 are defined for $-\infty < x < \infty$, and y_3 is defined for $0 < x < \infty$.

<u>announcement</u> The Science Learning Center offers study groups for Math 156 students. Online registration begins on Wednesday Sept 11 at 12pm via www.lsa.umich.edu/slc and groups start meeting on Sunday Sept 15. If the group you want is filled, please join the waitlist and another group may be opened.