1. Show that \( \int_a^b (f(x) + g(x)) \, dx = \int_a^b f(x) \, dx + \int_a^b g(x) \, dx \) using Riemann sums.

2. Show that \( \sum_{i=1}^{n} i^3 = \left( \frac{n(n+1)}{2} \right)^2 \) using two methods, (a) and (b), as indicated below.

   a) Use a telescoping sum as in class.

   b) Consider the square on the right. Each side is divided into segments of length \( \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, ..., \frac{1}{n} \), so that the length of each side is \( \frac{n(n+1)}{2} \). Now consider the regions \( a_1, a_2, a_3, ..., a_n \), where \( a_1 \) is a square with side 1, and region \( a_i \) for \( i = 2, 3, ..., n \) is L-shaped. Show that the area of region \( a_i \) is \( i^3 \). Finally note that the area of the square can be computed two ways - (1) squaring the length of a side, (2) summing the areas of regions \( a_1, a_2, a_3, ..., a_n \).

3. Evaluate \( \int_0^1 x^3 \, dx \) two ways.

   a) Riemann sums
   b) FTC

4. Evaluate \( \int_a^b x \, dx \) by Riemann sums.

5. a) Express \( \lim_{n \to \infty} \frac{1}{n} \sum_{i=1}^{n} \frac{1}{(i/n)^2} \) as an integral.
   b) Find the derivative of \( f(x) = \int_0^x \sqrt{1+t^3} \, dt \).

6. Evaluate by any means.
   a) \( \lim_{n \to \infty} \frac{1}{n} \left( \sqrt{\frac{1}{n}} + \sqrt{\frac{2}{n}} + \sqrt{\frac{3}{n}} + \cdots + \sqrt{\frac{n}{n}} \right) \)
   b) \( \int_1^4 \frac{dx}{\sqrt{x}} \)

7. A metal rod of length \( L \) m and cross-sectional area \( A \) m\(^2\) has mass density \( \rho(x) \) kg/m\(^3\), where \( x \) is measured in meters (m) from one end of the rod. (a) Find an expression for the total mass of the rod. (b) Compute the total mass for the case \( L = 4 \) m, \( A = 1 \) m\(^2\), \( \rho(x) = 9 + 2 \sqrt{x} \) kg/m\(^3\).

8. a) Derive the formula for the sum of a finite geometric series,
   \[ \sum_{i=0}^{n} r^i = 1 + r + r^2 + \cdots + r^n = \frac{1 - r^{n+1}}{1 - r}, \text{ if } r \neq 1. \]
   (hint: check the formula for \( n = 0, 1, 2 \) and then show it is true in general.)

   b) A student obtains a $1,000 loan and repays 50% of the balance each year, i.e. $500 is repaid in year 1, $250 is repaid in year 2, and so on. Express the total amount repaid after 10 years as a series and evaluate it using part (a).

   c) Evaluate \( \int_0^1 e^x \, dx \) using Riemann sums. (this completes problem 5c from hw1)

   d) What is the formula for the sum in part (a) if \( r = 1? \)

9. Consider the integral \( I = \int_0^1 e^{-x} \, dx = 1 - e^{-1} = 0.63212056 \). Let \( R_n, M_n \) be the right-hand and midpoint Riemann sums with \( n \) intervals. Construct a table as follows (use a calculator). column 1: \( n \) (take \( n = 1, 2, 4 \)); column 2: \( \Delta x \); column 3: \( R_n \); column 4: \( |I - R_n| \); column 5: \( M_n \); column 6: \( |I - M_n| \). For a given value of \( n \), which method gives a more accurate answer? When \( \Delta x \) decreases by 1/2, by what factor does the error decrease for each method?

**announcement**

The Science Learning Center offers study groups for Math 156 students. Check the SLC website www.lsa.umich.edu/slc for information. Online registration begins on Wednesday Sept 16 at 2pm and study groups begin meeting on Sunday Sept 20.