Math 156 Applied Honors Calculus II Fall 2019

hw3 , due: Tuesday, September 24

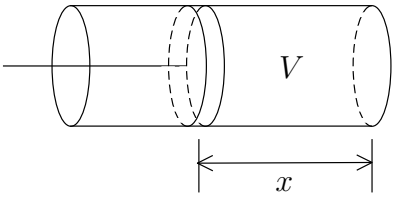
Please write neatly, explain the steps, express the answer in physical units when appropriate.

1. Find the work done in raising a 60 kg mass to a height of 2 m above the floor.

2. A spring has natural length 20 cm and a 25 N force is needed to stretch it to length 30 cm. Find the work done in stretching the spring from length 20 cm to 25 cm.

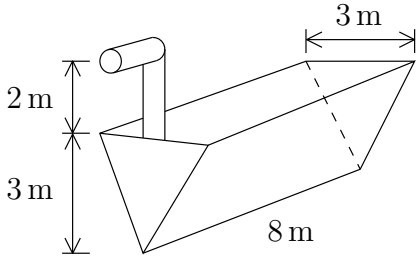
3. A 50 ft cable weighing 0.5 lb/ft hangs from the top of a building 120 ft high. a) Find the work done in pulling the cable to the top of the building. b) Find the work done in pulling half the cable to the top of the building.

4. a) A volume of compressed gas inside a closed cylinder expands as a piston is withdrawn. The gas pressure is a function of the gas volume, $P = P(V)$, and the force exerted by the gas on the piston is the product of the piston surface area and the gas pressure, $f = \pi R^2 P$, where R is the cylinder radius. Show that the work done when the gas expands from volume V_1 to V_2 is $W = \int_{V_1}^{V_2} P(V)dV$. (hint: start from $W = \int_a^b f(x)dx$, where x is the piston displacement and $f(x)$ is the force exerted by the gas on the piston)



b) In a steam engine, the steam pressure P and volume V satisfy the relation $PV^{1.4} = k$, where k is a constant. Use part (a) to calculate the work done by the engine during a cycle when the steam starts at pressure 1600 lb/in^2 and volume 100 in^3 and expands to volume 800 in^3. Express the answer in ft-lb.

5. A tank with the indicated shape is full of water. Find the work done in pumping the water to the top of the outlet. Express the answer in MJ. (hint: follow the steps in the example from class)



6. a) Sketch the graphs of $y_1 = \dfrac{1}{x}$ and $y_2 = \dfrac{x}{x^2+1}$ for $x \geq 0$ on the same plot, and label each curve.

b) Show that $\dfrac{x}{x^2+1} \sim \dfrac{1}{x}$ as $x \to \infty$.

7. True or False? Justify your answer.

a) If $\int_a^b f(x)\,dx \leq \int_a^b g(x)\,dx$, then $f(x) \leq g(x)$ for all x in the interval $[a, b]$.

b) The area under the graph of $y = \frac{1}{\sqrt{x}}$ from $x = 0$ to $x = 1$ is equal to two.

c) The integral $\int_0^1 \frac{dx}{2x-1}$ is an example of an improper integral.

8. The error function, defined by $\text{erf}(x) = \frac{2}{\sqrt{\pi}} \int_0^x e^{-t^2} dt$, is used in physics and probability.

Find the following quantities. Compute the value in (c) using the midpoint rule with $n = 2$.

a) $\text{erf}(0)$ b) $\text{erf}'(0)$ c) $\text{erf}(1)$ d) $\text{erf}'(1)$ e) $\lim_{x\to\infty} \text{erf}'(x)$

f) Sketch the graph of $\text{erf}(x)$ for $x \geq 0$. (hint: you may use the fact that $\int_0^\infty e^{-x^2} dx = \frac{\sqrt{\pi}}{2}$)

announcement On Friday Sept 20 we will meet in room B727 in the basement of East Hall for a computer lab using Maple, a software package for calculus and graphics.