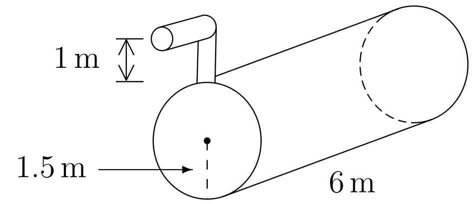
Math 156 Applied Honors Calculus II Fall 2018

hw4 , due: Tuesday, October 2

1. A cylindrical tank full of water is lying on its side. Find the work done in pumping the water to the top of the outlet. Use $\rho = 1000\,\mathrm{kg/m^3}$ for the water density. (hint: follow the steps from the example in class)



2. An object of mass m is moving in a straight line subject to a force $F(x)$, where x is the object's position. Let $v(x)$ be the object's velocity as a function of position.

a) Show that the work done in moving the object from x_0 to x_1 is equal to the change in the object's kinetic energy, $W = \int_{x_0}^{x_1} F(x)dx = \frac{1}{2}mv_1^2 - \frac{1}{2}mv_0^2$, where $v_0 = v(x_0), v_1 = v(x_1)$. (hint: $F(x) = ma = m\frac{dv}{dt}$, where a is the object's acceleration and t is time, then by the chain rule, $\frac{dv}{dt} = \frac{dv}{dx}\frac{dx}{dt} = \frac{dv}{dx}v$)

b) How many foot-pounds of work does it take to pitch a baseball at 90 mi/hr? Assume the baseball weighs 5 oz. (hint: 1 lb = 16 oz, 1 mile = 5280 ft, $g = 32\,\mathrm{ft/sec^2}$, 1 hr = 3600 sec; also recall that weight = force = mass $\times$ acceleration)

3. Consider the following integrals. a) $\int_{-\infty}^{\infty} xe^{-x^2}dx$ b) $\int_{-\infty}^{\infty} e^{-|x|}dx$ c) $\int_0^1 \frac{dx}{x\sqrt{x}}$

In each case sketch the function and determine whether the integral converges or diverges; if it converges, find the value.

4. Use the comparison theorem to show that $\int_1^{\infty} \frac{x}{\sqrt{1+x^6}}\,dx$ converges.

5. The speed of gas molecules at equilibrium is a random variable v, and the average speed is $\bar{v} = \frac{4}{\sqrt{\pi}}\left(\frac{M}{2RT}\right)^{3/2}\int_0^{\infty} v^3 e^{-Mv^2/(2RT)}dv$, where M is the molecular weight of the gas, R is the gas constant, and T is the gas temperature. Show that $\bar{v} = \sqrt{\frac{8RT}{\pi M}}$. (hint: substitute $x = Mv^2/2RT$)

6. The <u>Gamma function</u> $\Gamma(x)$ is defined by $\Gamma(x) = \int_0^{\infty} t^{x-1}e^{-t}dt$.

a) Evaluate $\Gamma(1),\ \Gamma(2),\ \Gamma(3)$.

b) Show that $\Gamma(n+1) = n\Gamma(n)$, for $n \geq 1$. Use this to compute $\Gamma(4)$.

Note that from (b) it follows that $\Gamma(n+1) = n!$ ("n factorial"), where $n! = n\cdot(n-1)\cdot(n-2)\cdots 2\cdot 1$.

7. Find the antiderivative. a) $\int \sin x \cos x\,dx$ b) $\int \sin^2 x \cos x\,dx$ c) $\int x\sqrt{x^2+a^2}\,dx$

8. Submit the problem from the Maple worksheet.

<u>announcement</u>

The 1st midterm exam is on Wednesday, October 3, 6:15-7:45pm, in CHEM 1400. If you have a conflict, please tell your instructor. The exam will cover: 1.1 sigma notation, 1.2 area, 1.3 definite integral, 1.4 FTC, 1.5 work, 1.6 improper integrals, 1.7 arclength, plus the homework and lecture notes. Calculators are not allowed on the exam. You may use one sheet of paper (one side) for handwritten notes. We will supply the exam booklets. A review sheet will be distributed before the exam.