Math 156 Applied Honors Calculus II Fall 2018

hw5 , due: Friday, October 12

1. The Laplace transform of a function $f(t)$ is a new function $F(s) = \int_0^\infty f(t)e^{-st}dt$; this construction is used in solving differential equations (Math 216/286/316). Find the Laplace transform $F(s)$ of the following functions. a) $f(t) = 1$ b) $f(t) = e^t$ c) $f(t) = t$

note : To ensure the integral converges, we assume $s > 0$ in (a,c) and $s > 1$ in (b).

2. Consider the integral $\int_0^\infty \left(\frac{x}{x^2+1} - \frac{c}{3x+1}\right)dx$, where $c > 0$ is a constant.

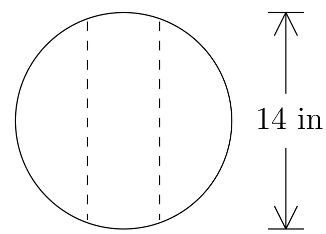
a) Show that evaluating it as $\int_0^\infty \frac{x}{x^2+1}dx - \int_0^\infty \frac{c}{3x+1}dx$ gives $\infty - \infty$, which is undefined.

b) Consider the functions $\frac{x}{x^2+1}$ and $\frac{c}{3x+1}$; for what value of c are they asymptotic to each other as $x \to \infty$?

c) Let c have the value found in (b), and evaluate the original integral by combining the two antiderivatives. In this way we make sense of $\infty - \infty$.

d) The value of the integral obtained in (c) is negative. Explain why that is reasonable.

3. Three students order a 14 inch pizza, and instead of slicing it the usual way, they slice it by two parallel cuts, at $x = a$ and $x = -a$. Find a formula for a ensuring that each student gets the same amount of pizza. Evaluate the integrals in the formula by the FTC, and solve for a using Maple (fsolve command) or a calculator. Express the answer in inches.



4. Find the antiderivative by the given method. These antiderivatives were derived in class by other methods; your current answers should be equivalent to those obtained in class.

a) $\int \sec\theta\, d\theta = \int \sec\theta \frac{\sec\theta + \tan\theta}{\sec\theta + \tan\theta}\, d\theta$, then substitute $u = \sec\theta + \tan\theta$

b) $\int \frac{du}{1-u^2} = \int \frac{1-u+u}{1-u^2}\, du = \int \frac{1-u}{1-u^2}\, du + \int \frac{u}{1-u^2}\, du = \int \frac{du}{1+u} + \int \frac{u\, du}{1-u^2} = \cdots$

5. The van der Waals equation of state of a gas gives the pressure P in terms of the volume V and temperature T as $P = \frac{RT}{V-b} - \frac{a}{V^2}$, where R is the ideal gas constant, and a, b are positive constants depending on the type of molecules in the gas. Note that when $a = b = 0$, the vdW formula reduces to the ideal gas law $PV = RT$. In an isothermal change of state, the temperature T is constant, and the work done in compressing the gas from volume V_1 to volume V_2 is $W = \int_{V_1}^{V_2} P dV$. Evaluate the integral and find W in terms of V_1, V_2, T, R, a, b.

6. Consider a circular sector with radius r and angle θ in the xy-plane. Let L be the arclength of the curved sector edge, and let A be the sector area. Show that $L = r\theta, A = \frac{1}{2}r^2\theta$, using the formulas for the arclength of a graph, $L = \int_a^b \sqrt{1+(f'(x))^2}\, dx$, and the area under a graph, $A = \int_a^b f(x)dx$. In each case you need to choose an appropriate $f(x), a, b$, and evaluate the formulas to obtain L, A in terms of r, θ. In the case of the area, write $A = A_1 + A_2$, where A_1 is the area of a triangle and A_2 is the area under the graph of a curve. (hint: in this problem θ is a given fixed parameter; when you apply trig substitution you must use a different symbol for the angle, e.g. ϕ)