Math 156 Applied Honors Calculus II Fall 2015

hw5, due: Friday, October 16

1. The Laplace transform of a function \( f(t) \) is a new function defined by \( F(s) = \int_0^\infty f(t)e^{-st}dt \). The Laplace transform is used in solving differential equations (Math 216/256). Find the Laplace transform \( F(s) \) of the following functions. 
   a) \( f(t) = 1 \) 
   b) \( f(t) = e^t \) 
   c) \( f(t) = t \)

   note: To ensure that the integral converges, we assume that \( s > 0 \) in (a,c), and \( s > 1 \) in (b).

2. Consider the integral \( \int_0^\infty \left( \frac{x}{x^2+1} - \frac{c}{3x+1} \right)dx \), where \( c > 0 \) is a constant.

   a) Show that the integral has the form \( \infty - \infty \) and hence in general it is undefined.

   b) Consider the functions \( \frac{x}{x^2+1} \) and \( \frac{c}{3x+1} \) as \( x \to \infty \). For what value of \( c \) are these functions asymptotic to each other?

   c) Let \( c \) be the value determined in (b) and evaluate the given integral in this case.

   d) The value of the integral obtained in (c) is negative. Explain why that is reasonable.

3. Three students order a 14 inch pizza, and instead of slicing it the usual way, they slice it by two parallel cuts, at \( x = a \) and \( x = -a \). Find a formula for \( a \) that ensures each student gets the same amount of pizza. Evaluate the integrals in the formula using the FTC. Solve the resulting formula for \( a \) using Maple (fsolve command) or a calculator.

4. Find the antiderivative by the given method. Note that these antiderivatives were derived in class by other methods; your current answers should be equivalent to those obtained in class.

   a) \( \int \sec \theta \, d\theta = \int \sec \theta \frac{\sec \theta + \tan \theta}{\sec \theta + \tan \theta} \, d\theta \), then substitute \( u = \sec \theta + \tan \theta \)

   b) \( \int \frac{du}{1-u^2} = \int \frac{1-u+u}{1-u^2} \, du = \int \frac{1-u}{1-u^2} \, du + \int \frac{u}{1-u^2} \, du = \int \frac{du}{1+u} + \int \frac{u \, du}{1-u^2} \), then integrate each term

5. The van der Waals equation of state of a gas gives the pressure \( P \) in terms of its volume \( V \) and temperature \( T \) as \( P = \frac{RT}{V-b} - \frac{a}{V^2} \), where \( R \) is the ideal gas constant and \( a, b \) are positive constants depending on the chemical composition of the gas molecules. Note that when \( a = b = 0 \), the vdW formula reduces to the ideal gas law \( PV = RT \). In an isothermal change of state, the temperature \( T \) is constant, and the work done in compressing the gas from volume \( V_1 \) to volume \( V_2 \) is given by \( W = \int_{V_1}^{V_2} P \, dV \). Evaluate the integral and find \( W \) in terms of \( V_1, V_2, T, R, a, b \).

6. Consider a circular sector of radius \( r \) and angle \( \theta \) in the \( xy \)-plane. Let \( L \) be the arclength of the curved edge of the sector, and let \( A \) be the area of the sector. Derive the formulas \( L = r\theta, A = \frac{1}{2}r^2\theta \) using the general formulas for the arclength of a graph, \( L = \int_a^b \sqrt{1 + (f'(x))^2} \, dx \), and the area under a graph, \( A = \int_a^b f(x) \, dx \). In each case you need to choose the appropriate \( f(x), a, b \), and then evaluate the general formulas to obtain \( L, A \) in terms of \( r, \theta \). In the case of the area, write \( A = A_1 + A_2 \), where \( A_1 \) is the area of a triangle and \( A_2 \) is the area under the graph of a curve.