1. The Laplace transform of a function \( f(t) \) is a new function \( F(s) = \int_0^\infty f(t)e^{-st}dt \); this construction is used in solving differential equations. Find the Laplace transform \( F(s) \) of the following functions.  
   a) \( f(t) = 1 \)  
   b) \( f(t) = e^t \)  
   c) \( f(t) = t \)  

   note: To ensure the integral converges, we assume \( s > 0 \) in (a,c) and \( s > 1 \) in (b).

2. Consider the integral \( \int_0^\infty \left( \frac{x}{x^2 + 1} - \frac{c}{3x + 1} \right) dx \), where \( c > 0 \) is a constant.

   a) Show that evaluating the integral as \( \int_0^\infty \frac{x}{x^2 + 1} dx - \int_0^\infty \frac{c}{3x + 1} dx \) gives \( \infty - \infty \), which is undefined.

   b) Consider the functions \( \frac{x}{x^2 + 1} \) and \( \frac{c}{3x + 1} \); for what value of \( c \) are they asymptotic to each other as \( x \to \infty \)?

   c) Let \( c \) have the value found in (b), and evaluate the integral by combining the two antiderivatives. In this way we make sense of the expression \( \infty - \infty \).

3. Three students order a 14 inch pizza, and instead of slicing it the usual way, they slice it by two parallel cuts, at \( x = a \) and \( x = -a \). Find a formula for \( a \) ensuring that each student gets the same amount of pizza. Evaluate the integrals in the formula by the FTC, and solve for \( a \) using Maple (fsolve command) or a calculator. Express the answer in inches.

4. Find the antiderivative by the given method. These antiderivatives were derived in class by other methods; your current answers should be equivalent to those obtained in class.

   a) \( \int \sec \theta \, d\theta = \int \sec \theta \frac{\sec \theta + \tan \theta}{\sec \theta + \tan \theta} \, d\theta \), then substitute \( u = \sec \theta + \tan \theta \)

   b) \( \int \frac{du}{1 - u^2} = \int \frac{1 - u + u}{1 - u^2} \, du = \int \frac{1 - u}{1 - u^2} \, du + \int \frac{u}{1 - u^2} \, du = \int \frac{du}{1 + u} + \int \frac{u \, du}{1 - u^2} \), then integrate each term

5. The van der Waals equation of state of a gas gives the pressure \( P \) in terms of the volume \( V \) and temperature \( T \) as \( P = \frac{RT}{V - b} - \frac{a}{V^2} \), where \( R \) is the ideal gas constant, and \( a, b \) are positive constants depending on the type of molecules in the gas. Note that when \( a = b = 0 \), the vdW formula reduces to the ideal gas law \( PV = RT \). In an isothermal change of state, the temperature \( T \) is constant, and the work done in compressing the gas from volume \( V_1 \) to volume \( V_2 \) is given by \( W = \int_{V_1}^{V_2} P \, dV \). Evaluate the integral and find \( W \) in terms of \( V_1, V_2, T, R, a, b \).

6. Consider a circular sector with radius \( r \) and angle \( \theta \) in the \( xy \)-plane. Let \( L \) be the arc length of the curved sector edge, and let \( A \) be the sector area. Show that \( L = r \theta, A = \frac{1}{2} r^2 \theta \), using the formulas for the arc length of a graph, \( L = \int_a^b \sqrt{1 + (f'(x))^2} \, dx \), and the area under a graph, \( A = \int_a^b f(x) \, dx \). In each case you need to choose appropriate \( f(x), a, b \), and evaluate the formulas to obtain \( L, A \) in terms of \( r, \theta \). In the case of the area, write \( A = A_1 + A_2 \), where \( A_1 \) is the area of a triangle and \( A_2 \) is the area under the graph of a curve. (hint: in this problem \( \theta \) is a given fixed parameter; when you apply trig substitution you must use a different symbol for the angle, e.g. \( \phi \))