1. a) A curve \( y = f(x), a \leq x \leq b, \) is rotated about the line \( y = c, \) where \( c \geq f(x). \) Find a formula for the resulting surface area. b) Find the area of the surface obtained by rotating the circle \( x^2 + y^2 = r^2 \) about the line \( y = r. \)

2. An observer is located at height \( h \) above the North pole of a sphere of radius \( r. \) Show that the observer can see a portion of the sphere with area \( S = \frac{2\pi rh}{r+h}. \) (hint: use the formula for the surface area generated by rotating a curve about an axis)

3. Three point masses with \( m_1 = 25, m_2 = 20, m_3 = 10 \) are located on the \( x \)-axis at \( x_1 = -2, x_2 = 3, x_3 = 7. \) Sketch the mass distribution and find the center of mass.

4. Four point masses with \( m_1 = 6, m_2 = 5, m_3 = 1, m_4 = 4 \) are located in the \( xy \)-plane at \( (x_1, y_1) = (1, -2), (x_2, y_2) = (3, 4), (x_3, y_3) = (-3, -7), (x_4, y_4) = (6, -1). \) Sketch the mass distribution and find the center of mass.

5. A triangular plate with uniform density has vertices \((-1, 0), (1, 0), (0, 2)\) in the \( xy \)-plane. Find the center of mass.

6. Sketch the region bounded by the curves \( y = 4 - x^2, y = 0 \) and find the center of mass. Assume \( \rho = 1. \)

7. Let \( f(t) \) be the pdf for the time it takes to drive to work, where \( t \) is measured in minutes. Express the following probabilities as integrals.
   a) the probability that the drive lasts less than 15 minutes
   b) the probability that the drive takes more than half an hour

8. Let \( f(x) = \frac{3}{64} x \sqrt{16 - x^2} \) for \( 0 \leq x \leq 4 \) and \( f(x) = 0 \) for all other values of \( x. \)
   a) Sketch \( f(x) \) for \(-\infty < x < \infty. \) b) Verify that \( f(x) \) is a valid pdf. c) Find \( \text{prob}(0 \leq X \leq 2). \)

9. a) Evaluate the quantity \( (1 + \frac{1}{n})^n \) for \( n = 1, 10, 10^2, 10^3, 10^4 \) using a calculator or Maple.
   b) Evaluate the limit \( \lim_{n \to \infty} (1 + \frac{1}{n})^n. \) (hint: use the fact that \( \ln e = 1)\)

10. This exercise introduces the hyperbolic functions.
   a) We define \( \cosh x = \frac{e^x + e^{-x}}{2}, \sinh x = \frac{e^x - e^{-x}}{2}. \) Show that \( \cosh x \) is an even function (i.e. \( f(-x) = f(x) \)) and \( \sinh x \) is an odd function (i.e. \( f(-x) = -f(x) \)).
   b) Sketch \( y = \cosh x \) and \( y = \sinh x \) on the same axes for \(-\infty < x < \infty. \)
   c) Show that \( \cosh^2 x - \sinh^2 x = 1. \)

Note: The result in (c) implies that the point \( (X, Y) = (\cosh x, \sinh x) \) lies on the hyperbola \( X^2 - Y^2 = 1 \) in the \( XY \)-plane, and this is why \( \cosh x, \sinh x \) are called hyperbolic trigonometric functions. The usual functions \( \cos x, \sin x \) are sometimes called circular trigonometric functions because they satisfy the equation \( \cos^2 x + \sin^2 x = 1, \) which implies that the point \( (X, Y) = (\cos x, \sin x) \) lies on the circle \( X^2 + Y^2 = 1. \) By analogy with the definitions \( \cosh x = \frac{e^x + e^{-x}}{2}, \sinh x = \frac{e^x - e^{-x}}{2}, \) it can be shown that \( \cos x = \frac{e^{ix} + e^{-ix}}{2}, \sin x = \frac{e^{ix} - e^{-ix}}{2i}, \) where \( i = \sqrt{-1}. \) We’ll discuss this later in the course as a consequence of Euler’s formula.

   d) Find \( \frac{d}{dx} \cosh x, \frac{d}{dx} \sinh x. \)
   e) Define \( \tanh x = \frac{\sinh x}{\cosh x}. \) Show that \( \tanh x \) is an odd function. Find \( \frac{d}{dx} \tanh x. \)
   f) Evaluate \( \lim_{x \to \pm \infty} \tanh x. \) Sketch the graph of \( \tanh x \) for \(-\infty < x < \infty. \)