Math 156 Applied Honors Calculus II Fall 2018

hw6 , due: Tuesday, October 23

1. a) A curve $y = f(x), a \le x \le b$, is rotated about the line $y = c$, where $c \ge f(x)$. Find a formula for the resulting surface area. b) Using part (a), find the area of the surface obtained by rotating the circle $x^2 + y^2 = r^2$ about the line $y = r$.

2. An observer is located at height h above the North pole of a sphere of radius r. (a) Show that the observer can see a portion of the sphere with area $S = \frac{2\pi r^2 h}{r+h}$. (hint: draw a picture, use the formula for the surface area obtained by rotating a curve about an axis) (b) Check the answer in two limits, $h \to 0$ and $h \to \infty$.

3. Sketch the mass distribution, find the center of mass, and indicate the CM on the sketch.

a) three point masses on the x-axis; $m_1 = 25, m_2 = 20, m_3 = 10, x_1 = -2, x_2 = 3, x_3 = 7$

b) four point masses in the xy-plane; $m_1 = 6, m_2 = 5, m_3 = 1, m_4 = 4, (x_1, y_1) = (1, -2), (x_2, y_2) = (3, 4), (x_3, y_3) = (-3, -7), (x_4, y_4) = (6, -1)$

4. Sketch the region in the xy-plane, find the center of mass, and indicate the CM on the sketch.

a) a triangular plate with vertices $(-1, 0), (1, 0), (0, 2)$

b) the region bounded by the curves $y = 4 - x^2,\ y = 0$

5. Let X be a random variable with pdf $f(x) = \frac{3}{64}x\sqrt{16 - x^2}$ for $0 \le x \le 4$ and $f(x) = 0$ for all other x. (a) Sketch the graph of $f(x)$. (b) Verify that $f(x)$ is a valid pdf. (c) Find prob$(0 \le X \le 2)$.

6. a) Compute $\left(1 + \dfrac{1}{n}\right)^n$ for $n = 1, 10, 10^2, 10^3, 10^4$ using a calculator or Maple.

b) Evaluate $L = \lim\limits_{n\to\infty} \left(1 + \dfrac{1}{n}\right)^n$. (hint: consider $\ln L$)

7. This exercise introduces the hyperbolic functions.

a) Define $\cosh x = \dfrac{e^x + e^{-x}}{2}$, $\sinh x = \dfrac{e^x - e^{-x}}{2}$. Show that $\cosh x$ is an even function (i.e. $f(-x) = f(x)$) and $\sinh x$ is an odd function (i.e. $f(-x) = -f(x)$).

b) Sketch $y = \cosh x$ and $y = \sinh x$ on the same plot for $-\infty < x < \infty$.

c) Show that $\cosh^2 x - \sinh^2 x = 1$.

This result shows that the point $(X, Y) = (\cosh x, \sinh x)$ lies on the hyperbola $X^2 - Y^2 = 1$ in the XY-plane, and this is why $\cosh x$, $\sinh x$ are called hyperbolic trigonometric functions. The familiar functions $\cos x$, $\sin x$ are sometimes called circular trigonometric functions because they satisfy the equation $\cos^2 x + \sin^2 x = 1$, which implies that the point $(X, Y) = (\cos x, \sin x)$ lies on the circle $X^2 + Y^2 = 1$.

d) Find $\frac{d}{dx}\cosh x$, $\frac{d}{dx}\sinh x$.

e) Define $\tanh x = \dfrac{\sinh x}{\cosh x}$. Show that $\tanh x$ is an odd function. Find $\frac{d}{dx}\tanh x$.

f) Evaluate $\lim\limits_{x\to\pm\infty} \tanh x$. Sketch the graph of $\tanh x$ for $-\infty < x < \infty$.

A final comment - by analogy with $\cosh x = \dfrac{e^x + e^{-x}}{2}$, $\sinh x = \dfrac{e^x - e^{-x}}{2}$, it can be shown that $\cos x = \dfrac{e^{ix} + e^{-ix}}{2}$, $\sin x = \dfrac{e^{ix} - e^{-ix}}{2i}$, where $i = \sqrt{-1}$; this is a consequence of Euler's formula; we'll see this later in the course.