Math 156 Applied Honors Calculus II Fall 2018

hw8 , due: Tuesday, November 6

1. The average waiting time in a fast-food restaurant is 2.5 minutes. a) Find the probability that a customer is served in the first 2 minutes. b) Find the probability that a customer waits 4 minutes or more. c) The manager wants to advertise that anyone waiting more than a certain amount of time will receive a free meal. What should the advertisement say to avoid giving free meals to more than 2% of the customers?

2. When the chemical reaction $N_2O_5 \to 2NO_2 + \frac{1}{2}O_2$ takes place at 45° C, the concentration of dinitrogen pentoxide satisfies the differential equation, $\frac{d}{dt}[N_2O_5] = -0.0005 \cdot [N_2O_5]$, where t is measured in seconds. a) Find an expression for the concentration of N_2O_5 after t seconds if the initial concentration is c_0. b) How long does it take for the N_2O_5 concentration to fall to 90% of its initial value?

3. a) How long does it take for an investment to double in value if the annual interest rate is 6% and the interest is compounded continuously? b) Find the equivalent annual interest rate.

4. A tank initially contains 1000 L of brine with 15 kg of dissolved salt, and pure water starts pouring into the tank at a rate of 10 L/min. The solution is well mixed, and it drains from the tank at the same rate the pure water enters. Find the amount of salt in the tank after (a) t minutes, (b) 20 minutes?

5. A lake is initially stocked with 400 fish and the population triples after one year. Based on the lake size and nutrients available, the maximum capacity of the lake is estimated to be 10,000 fish. a) Using the logistic equation, find an expression for the fish population after t years. b) How long will it take for the fish population to reach 5,000?

6. Consider the differential equation $y' = -y$ with initial condition $y_0 = 1$. In this problem you will solve the equation numerically for $y(1)$ using Euler's method with $t_n = n\Delta t = 1$. Make a table with the following entries. column 1: Δt (time step, take $\Delta t = 1, \frac{1}{2}, \frac{1}{4}, \frac{1}{8}$); column 2: u_n (numerical solution at time t_n); column 3: $|y_n - u_n|$ (error); column 4: $|y_n - u_n|/\Delta t$ (ratio of error to time step) a) Indicate the limit of each column as in class. b) If the time step Δt is reduced by a factor of $\frac{1}{2}$, by approximately what factor is the error reduced?

7. a) Show that $\sinh^{-1}x = \ln\left(x + \sqrt{x^2+1}\right)$. (hint: set $x = \sinh y$ and solve for y)

b) In class we showed that $\int \frac{dx}{\sqrt{x^2+1}} = \ln\left(x + \sqrt{x^2+1}\,\right)$ using the trig substitution $x = \tan\theta$. Now derive this result using the hyperbolic trig substitution $x = \sinh y$.

8. Given a function $f(x)$ and a point $x = a$, define $T_1(x) = f(a) + f'(a)(x-a)$; it is a linear function of x and it is called the <u>Taylor polynomial of degree 1</u>.

a) Show that $T_1(a) = f(a), T_1'(a) = f'(a)$. Hence $T_1(x)$ is tangent to $f(x)$ at $x = a$ and we view it as a <u>linear approximation</u> to $f(x)$ at $x = a$.

b) Show that $f(x) = f(a) + f'(a)(x-a) + \int_a^x (x-t)f''(t)\,dt$. (hint: start from the integral and apply integration by parts with $u = x - t$, $dv = f''(t)\,dt$.)

c) Using part (b), derive the error bound $|f(x) - T_1(x)| \le \frac{1}{2}M_2|x-a|^2$, where $M_2 = \max|f''(t)|$. (hint: follow the steps from problem 10 on hw7).

d) Let $f(x) = e^x, a = 0$. Find $T_1(x)$. Sketch $f(x), T_1(x)$ on the same graph around $x = a$.

e) Make a table with the following format. column 1: $|x-a|$, column 2: $|f(x) - T_1(x)|$. Take $f(x) = e^x, a = 0$ and fill in the entries for $x = 1, 1/2, 1/4, 1/8$ using a calculator. When $|x-a|$ is reduced by a factor of $\frac{1}{2}$, by what factor is the error $|f(x) - T_1(x)|$ reduced?