Math 156 Applied Honors Calculus II Fall 2018

hw11 , due: Wednesday, December 5

1. Consider the power series $1+\frac{(x-1)}{2}+\frac{(x-1)^2}{2^2}+\frac{(x-1)^3}{2^3}+\cdots=\sum_{n=0}^{\infty}\frac{(x-1)^n}{2^n}$.

a) Find the function $f(x)$ represented by the series. b) Find the interval of convergence of the series. c) Sketch the graph of $f(x)$ and indicate the interval of convergence on the x-axis.

2. Find $T_1(x)$ and $T_3(x)$, the 1st and 3rd degree Taylor polynomials for $f(x)=\sin x$ at the point $x=\pi$. Sketch the graphs of $f(x), T_1(x), T_3(x)$ on the same plot.

3. Express the integral $\int_0^{1/2}\frac{dx}{1+x^6}$ as a series. Give the first three terms and the general term. How many terms are needed to ensure the error is less than 10^{-5}? Use the error bound derived in class.

4. We can approximate $\sqrt{10}$ using the fact that it is close to $\sqrt{9}=3$.

(a) Find $T_1(x)$, the 1st degree Taylor approximation for $f(x)=\sqrt{x}$ at $a=9$, and evaluate $T_1(10)$.

(b) Repeat for $T_2(x)$, the 2nd degree Taylor approximation.

Express the answers as a rational number (i.e. $\frac{m}{n}$, where m,n are integers) and in decimal form with eight digits. Then find $\sqrt{10}$ using a calculator. Do the results become more accurate as the degree of the Taylor approximation increases?

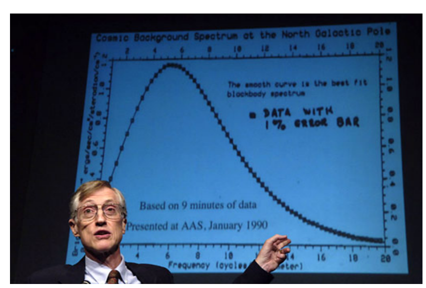
5. A non-reflecting object that absorbs all incident radiation is called a blackbody. When a blackbody is held in thermal equilibrium, it emits radiation with energy density $f(\lambda)$, where λ is the radiation wavelength. The Rayleigh-Jeans law for the energy density, proposed in the late 1800s, is $f_{RJ}(\lambda)=\frac{8\pi kT}{\lambda^4}$, where T is the object's temperature, and k is the Boltzmann constant. The Rayleigh-Jeans law agrees with experiments for long wavelength ($f_{RJ}(\lambda)\to 0$ for $\lambda\to\infty$), but it disagrees with experiments for short wavelength ($f_{RJ}(\lambda)\to\infty$ for $\lambda\to 0$, while experiments show that $f(\lambda)\to 0$ for $\lambda\to 0$). In 1900 Max Planck applied the novel idea of energy quantization to derive a different expression, $f_P(\lambda)=\frac{8\pi hc\lambda^{-5}}{e^{hc/\lambda kT}-1}$, where h is Planck's constant and c is the speed of light.

a) Show that $\lim_{\lambda\to 0} f_P(\lambda)=0$ and $\lim_{\lambda\to\infty} f_P(\lambda)=0$; this means that Planck's law agrees with experiments in the limits of short wavelength and long wavelength. (hint: set $x=hc/\lambda kT$)

b) Show that $f_P(\lambda)\sim f_{RJ}(\lambda)$ for $\lambda\to\infty$; recall that this is equivalent to $\lim_{\lambda\to\infty}\frac{f_P(\lambda)}{f_{RJ}(\lambda)}=1$.

c) Show that $f_P(\lambda)<f_{RJ}(\lambda)$ for $\lambda\geq 0$.

d) Sketch $f_P(\lambda)$ and $f_{RJ}(\lambda)$ for $\lambda\geq 0$ on the same plot.



Planck's discovery is explained in the article, "Max Planck: the reluctant revolutionary"; a link is on the course website. In 2006, John Mather and George Smoot won the Nobel Prize in Physics, "for their discovery of the blackbody form ... of the cosmic microwave background radiation"; the photo shows John Mather with a plot of the experimental data in excellent agreement with Planck's law.

announcement

The final exam is on Friday, Dec 14 at 10:30am-12:30pm in Angell Hall Auditorium D. The exam covers the entire course; calculators are not allowed; you may use two sheets of notes (e.g. two sides of one page); we will supply the exam booklets; a review sheet will be distributed soon.