Math 156 Applied Honors Calculus II Fall 2011

hw11 , due: Wednesday, December 7

Find the radius of convergence and interval of convergence of the series.

789/4 $\sum_{n=0}^{\infty} \frac{(-1)^n x^n}{n+1}$ 789/10 $\sum_{n=1}^{\infty} \frac{x^n}{n3^n}$

789/29 If $\sum_{n=0}^{\infty} c_n 4^n$ converges, do the following series also converge?

a) $\sum_{n=0}^{\infty} c_n(-2)^n$ b) $\sum_{n=0}^{\infty} c_n(-4)^n$

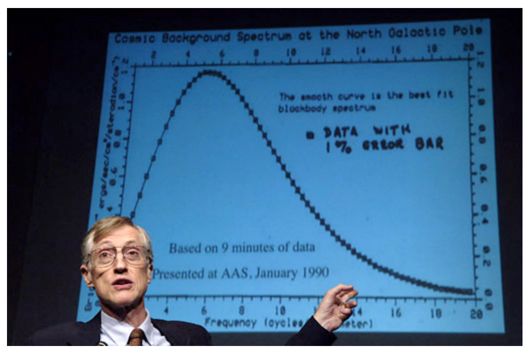
795/3 Find the power series representation of $f(x) = \frac{1}{1+x}$ about $x = 0$ and determine the interval of convergence.

795/30 Use a power series to approximate $\int_0^{1/2} \frac{dx}{1+x^6}$ to six decimal places.

795/32 Show that $f(x) = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n}}{(2n)!}$ is a solution of the differential equation $y'' + y = 0$.

821/1, 2, 3

These problems concern the formula for the energy density of blackbody radiation, $f(\lambda)$. (hints: for 821/1, set $x = \frac{hc}{\lambda kT}$; for 821/2, use the 1st order Taylor approximation for e^x about $x = 0$; for 821/3, sketch the functions for $\lambda \geq 0$; you may check your sketch using Maple or a graphing calculator)



As described on page 821, the Rayleigh-Jeans formula for $f(\lambda)$ was corrected by Max Planck in 1900 using the novel idea of energy quantization (the full story is explained in the article, "Max Planck: the reluctant revolutionary"; the course website has a link to the article). In 2006, John Mather and George Smoot won the Nobel Prize in Physics, "for their discovery of the blackbody form and anisotropy of the cosmic microwave background radiation", which confirms the big bang theory of the origin of the Universe. The photo shows John Mather with a plot of the experimental microwave data for $f(\lambda)$; the data curve agrees very well with Planck's formula for $f(\lambda)$, which you are asked to sketch in problem 821/3.

1. Find an approximate value for $\sqrt{10}$ using the idea that $\sqrt{10}$ should be close to $\sqrt{9} = 3$. Let $f(x) = \sqrt{x}$ and approximate $\sqrt{10}$ by $T_1(10)$, where $T_1(x)$ is the 1st degree Taylor polynomial for $f(x) = \sqrt{x}$ about $x = 9$. Repeat for the 2nd and 3rd degree Taylor polynomials. Also find $\sqrt{10}$ using a calculator. Do the Taylor approximations become more accurate as the order increases?

announcements

1. The final exam is on Thursday, December 15, 8-10am, room tba. The exam will cover the entire course. Calculators are not allowed on the exam. You may use two sheets of notes (e.g. two sides of one page). We will supply the exam booklets.

2. The online course evaluations are available from Friday Dec 2 to Wednesday Dec 14. Please complete the evaluations - they provide valuable feedback from students to instructors.