Math 371 **Fall 2008** **Homework 1** **due: Tues Sept 16**

section 1.3, floating point number systems

1. (a) Convert $(2008)_{10}$ to base 2. (b) Convert $(1111.11)_2$ to base 10.

2. The floating point representation of a real number has the form $\pm(0.d_1 d_2 \dots d_n)_\beta \cdot \beta^e$, where $d_1 \neq 0$ and $-M \leq e \leq M$. Consider a system with $\beta = 2$, $n = 4$, and $M = 5$.

(a) Find the smallest and largest positive numbers that can be represented in this system. Give your answers in decimal form.

(b) Find the floating point number in this system that is closest to $\sqrt{2}$.

3. page 40, problem 8(a) (read "An Example to Set the Stage" on page 31)

section 1.4, finite precision arithmetic

4. page 52, problem 13

5. Consider the equation $x^2 + 25x + 0.1 = 0$.

(a) Solve for the roots using the quadratic formula, $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$. Use 4-digit arithmetic (as in the example from class). Compare with the values obtained using Matlab.

(b) Repeat using the alternative form of the quadratic formula, $x = \dfrac{2c}{-b \pm \sqrt{b^2 - 4ac}}$. Compare with the results of part (a) and explain your observations.

6. Let $f(x)$ be a given function and recall the forward difference approximation of $f'(x)$,

$$D_+ f(x) = \frac{f(x+h) - f(x)}{h},$$

where $h > 0$ is the step size.

(a) Take $f(x) = \sin x, x = \pi/4, h = 2^{-n}$ for $n = 1, 2, \dots, 6$. Following the example in class, plot the error versus h and make a table with the following format: column 1: h, column 2: $D_+ f(x)$, column 3: $f'(x) - D_+ f(x)$, column 4: $(f'(x) - D_+ f(x))/h$, column 5: $(f'(x) - D_+ f(x))/h^2$, column 6: $(f'(x) - D_+ f(x))/h^3$. You may modify the Matlab code given in class. Present at least eight decimal places (type "format long" in Matlab to get the full 15 digits).

(b) Repeat for the centered difference approximation,

$$D_0 f(x) = \frac{f(x+h) - f(x-h)}{2h},$$

which also approximates $f'(x)$. Which approximation is more accurate? Explain why.