

## chapter 3, linear algebra

1. Which of the following matrices are invertible? Justify your answer.

(a)  $\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$  (b)  $\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$  (c)  $\begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix}$  (d)  $\begin{pmatrix} 1 & 0 & 2 \\ -1 & 3 & 1 \\ 0 & 3 & 3 \end{pmatrix}$

2. page 157, problem 2 (Gaussian elimination) - omit, duplicate from hw2

3. page 169, problem 14 (i) (no pivoting), (ii) (partial pivoting)

4. page 180, problems 1, 2a (vector norms)

5. Let  $A = \begin{pmatrix} 1 & 1 \\ 2 & 0 \end{pmatrix}$ .

(a) Find  $\frac{\|Ax\|_\infty}{\|x\|_\infty}$  for the following vectors  $x$ .  $x_1 = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$ ,  $x_2 = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$ ,  $x_3 = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$ .

(b) Find a vector  $x$  such that  $\frac{\|Ax\|_\infty}{\|x\|_\infty} = \|A\|_\infty$ .

(c) Repeat part (a) for the 2-norm.

(d) (optional) Repeat part (b) for the 2-norm.

6. Let  $A = \begin{pmatrix} 1.2969 & 0.8648 \\ 0.2161 & 0.1441 \end{pmatrix}$ ,  $b = \begin{pmatrix} 0.8642 \\ 0.1440 \end{pmatrix}$ ,  $x = \begin{pmatrix} 2 \\ -2 \end{pmatrix}$ ,  $x_1 = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$ ,  $x_2 = \begin{pmatrix} 0.9911 \\ -0.4870 \end{pmatrix}$ .

a) Show that  $x$  is the exact solution of  $Ax = b$ .

b) Think of  $x_1, x_2$  as approximations to the exact solution  $x$ . Compute the errors  $e_1, e_2$  and corresponding residuals  $r_1, r_2$ .

c) Find  $\|A\|_\infty$ ,  $\|A^{-1}\|_\infty$ ,  $\kappa_\infty(A)$ .

d) In class we proved the following theorem relating the relative error, relative residual, and condition number.

$$\frac{\|e\|}{\|x\|} \leq \kappa(A) \frac{\|r\|}{\|b\|}$$

Check that this result holds for the approximate solutions  $x_1, x_2$  given above.

7. Derive the following result, which was stated in class.

$$\left. \begin{array}{l} Ax = b \\ \tilde{A}\tilde{x} = b \end{array} \right\} \Rightarrow \frac{\|x - \tilde{x}\|}{\|\tilde{x}\|} \leq \kappa(A) \frac{\|A - \tilde{A}\|}{\|A\|}$$

note : This says that in solving a linear system  $Ax = b$ , the condition number controls the size of perturbations in the solution due to perturbations of the matrix.

### announcements

The 1st midterm exam will be on Thursday Oct 16 in class. A review sheet with sample problems will be distributed before the exam. Calculators are allowed, although the problems won't be heavily computational and will instead test understanding. You may use one sheet of notes (i.e. one side of one page, 8.5 in  $\times$  11 in). Exam booklets will be provided.

The 2nd midterm exam will be on Thursday Nov 20 in class.