**Math 371** **Fall 2008** **Homework 4** **due: Thurs Nov 6**

**section 3.7, special matrices**

1. Show that the following matrices are positive definite.

(a) $\begin{pmatrix} 2 & 1 \\ 1 & 2 \end{pmatrix}$ (b) $\begin{pmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 2 \end{pmatrix}$

**section 3.8, iterative methods**

2. Consider the linear system, $2x_1 + x_2 = 1, x_1 + 2x_2 = -1$.

(a) Write the system in matrix form and solve it by LU factorization.

(b) Write out Jacobi's method in component form and take three steps starting from initial guess $x_0 = (0,0)^T$. Present the results in a table with the following format.
column 1: $k$ (iteration step)
column 2: $x_k = (x_1^{(k)}, x_2^{(k)})^T$ (solution vector at step $k$)
column 3: $||e_k||_\infty$ (error norm at step $k$)
column 4: $||e_k||_\infty / ||e_{k-1}||_\infty$ (ratio of successive error norms at step $k$)
Find the iteration matrix $B_j$ and compute $||B_J||_\infty, \rho(B_J)$. Does the method converge?

(c) Repeat part (b) for Gauss-Seidel.

(d) Repeat part (b) for optimal SOR.

3. Find the e-values and e-vectors of the following matrices. Do this by hand. (Check your answers using Matlab - type `help eig` to learn the command.)

a) $\begin{pmatrix} 2 & 0 \\ 0 & 1 \end{pmatrix}$ b) $\begin{pmatrix} 0 & 2 \\ 1 & 0 \end{pmatrix}$ c) $\begin{pmatrix} 2 & -1 \\ -1 & 2 \end{pmatrix}$ d) $\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$

**Computing Project 1 , due: Thurs Nov 13**

**section 8.1, two-point boundary value problem**

page 672, problem 15a (deflection of a beam)

This problem asks you to solve a two-point boundary value problem for the deflection of a wooden beam. The deflection of the beam's centerline, $u(x)$, satisfies the problem

$$u'' - \frac{T}{EI}u = -\frac{w}{2EI}x(L-x) \quad , \quad u(0) = u(L) = 0.$$

Just do part (a), in which the beam cross-section is constant. The textbook asks you to compute the beam deflection at 1-inch intervals, but you should do two additional cases, 4-inch and 2-inch intervals (so we can assess the accuracy of the results). Use Matlab. Set up the finite-difference scheme and solve the linear system using the tridiagonal $LU$ method discussed in class. Plot all three cases in a single plot (use different symbols or colors to distinguish the cases).