Math 371 **Fall 2008** **Homework 5** **due: Thurs Dec 4**

chapter 4, computing eigenvalues

1. In class we considered the e-value problem for the matrix $A = \begin{pmatrix} 2 & 1 & 1 \\ 1 & 3 & 1 \\ 1 & 1 & 4 \end{pmatrix}$ (see lecture notes, chapter 4, page 5). The largest e-value is $\lambda_1 = 5.214319743377535$ (and this can be confirmed using the Matlab `eig` command). A table of results was presented for the power method, shifted inverse iteration with $\mu = 5$, and Rayleigh quotient iteration. The initial guess for the e-vector was $v^{(0)} = \frac{1}{\sqrt{3}} \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$. The computed e-values $\lambda^{(k)}$ were given for iterations $k = 0, 1, 2$. Repeat the calculations and extend the table to $k = 3$. Use Matlab and present 15 decimal digits for the computed e-values (type `format long`). For each method, underline the correct digits at the final step $k = 3$.

chapter 5, approximation and interpolation

2. a) Find the Taylor series of $f(x) = \sin x$ about $x = 0$.

b) Using Matlab, plot the Taylor polynomials $p_n(x)$ of degree $n = 1, 3, 5, 7$ on the same plot on the interval $-4\pi \le x \le 4\pi$. Also plot the original function $f(x) = \sin x$. Label each curve. We view the Taylor polynomial as an approximation to the original function. Describe in words what happens to the approximation as the degree of the polynomial increases.

Computing Project 2 , due: Tues Dec 9

chapter 9, two-dimensional boundary value problems

Consider the problem of finding the temperature distribution on a metal plate. Assume that the plate is given by the unit square $0 \le x, y \le 1$. The temperature $\phi(x, y)$ satisfies the Laplace equation $\phi_{xx} + \phi_{yy} = 0$ on the square with boundary conditions $\phi(x, 1) = 1, \phi(x, 0) = \phi(0, y) = \phi(1, y) = 0$. In physical terms, this means that there are no heat sources in the plate, and one side of the plate is heated to a constant temperature while the other three sides are kept at a lower reference temperature. Suppose the problem is solved using the standard finite-difference scheme $D_+^x D_-^x w_{ij} + D_+^y D_-^y w_{ij} = 0$ with mesh size $h = \frac{1}{n+1}$. This yields a linear system denoted by $A_h w_h = f_h$, where $w_h = \{w_{ij}\}$ is the numerical solution vector with components $w_{ij} \approx \phi(x_i, y_j)$, and the mesh points are given by (x_i, y_j) with $x_i = ih, y_j = jh, i, j = 0 : n + 1$. Use Matlab to solve the linear system by Jacobi's method with mesh sizes $h = \frac{1}{4}, \frac{1}{8}, \frac{1}{16}$. Let $w_h^{(0)} = 0$ be the starting vector. Take $||r_k||_\infty \le 10^{-2}$ for the stopping criterion, where $r_k = A_h w_h^{(k)} - f_h$ is the residual at step k. To keep the code simple, the numerical solution $w_h = \{w_{ij}\}$ should be coded as a matrix of dimension $(n+2) \times (n+2)$ containing the unknown interior temperature values and the known boundary values. Present the results as follows. Include a copy of the code.

a) For each value of h, plot the computed temperature w_{ij} at the final step using a contour plot and a mesh plot (type `help contour` and `help mesh` in Matlab for instructions).

b) For each value of h, print the number of iterations required to reach the stopping criterion.

c) Give a brief writeup describing the results.

(hint : compare your results with the lecture notes, chapter 3, page 26, to see if you're on the right track)